Analysis of Transit Time Effects on Doppler Flow Measurement

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Abstract—A relation is derived between the transmitted spectral density and the output spectral density of broad bandwidth, random signal, Doppler flow measurement systems operating under conditions where the fluid transit time is limited by the transmitted signal bandwidth and not by beam geometry. The fact that this result is already known to hold for pulsed radio frequency (RF) Doppler proves that random or pseudorandom Dopplers will give the same output spectrum as pulsed RF Dopplers, provided that the transmitted spectral density of the RF system has the same envelope as that of the noise system.

Doppler output spectra for several cases important in blood flow measurement are calculated and the results are confirmed experimentally. A correction factor is derived for Doppler flow velocity estimates in regions of large velocity gradient, and the relation between average flow velocity in a tube and the first moment of the Doppler spectrum is calculated for the case of a finite bandwidth transmitted signal. It is pointed out that the technique introduced here will make it possible to calculate the effects on the Doppler output spectrum of beam geometry, Rayleigh scattering and frequency dependent absorption.

INTRODUCTION

ULTRASONIC Doppler flowmeters which transmit a pulsed deterministic signal are at present mainly confined to blood flow measurement because of their severely limited range [1]-[4]. However, the introduction of pseudorandom code systems [5] which have a less limited range, and of random signal systems [6] whose range is unlimited, promise to broaden the uses of ultrasonic Doppler flowmeters to applications such as wake measurements in turbid water, etc., etc.

A relation is derived below between the output power density spectrum of a random signal ultrasonic Doppler system viewing random scatterers such as blood cells or air bubbles in water, and the power density spectrum of the transmitted signal. This relation turns out to be identical with a relation derived by Altes [7] relating the transmitted and output energy spectral densities for pulsed or continuous radio frequency (RF) Doppler systems observing uniformly distributed moving scatterers such as erythrocytes in blood. This identity proves that a random or pseudorandom code Doppler system illuminating a particular flow will produce an output spectral power density having the same envelope as that produced by an RF Doppler system whose transmitted spectral energy density has the same envelope as that of the power spectral density of the random or pseudorandom signal system. Experimental evidence of this has recently been published [8].

When making high-resolution blood flow measurements in the current state of the art, the frequency f_d of the peak energy of the Doppler output spectrum is substituted into the conventional Doppler equation

$$f_d = \frac{2v}{c} f_o \cos\theta \tag{1}$$

to estimate the velocity v in the region of observation.

In addition, average velocity estimates in a blood vessel are often made [9], [10], using the normalized first moment of the single-sided Doppler output spectrum $S_z(\omega)$, i.e.,

$$\overline{F_d} = \frac{2\bar{v}}{c}\cos\theta = \frac{\int_0^\infty \omega S_z(\omega)\,d\omega}{\int_0^\infty S_z(\omega)\,d\omega}. \tag{2}$$

In the analysis leading to (1) and (2) both the finite bandwidth of the transmitted signal as well as the effects of the finite transit time of the scatterers through the flowmeter system range cell are neglected. The range cell is defined in this paper as the distance along the ultrasound beam over which the measured velocity is averaged. For a pulsed RF system the range cell equals half the spatial length of an emitted RF burst at any one instant. For a random signal system the range cell is that region from which echoes coherent with the reference signal are returned to the receiver [8]. The reference signal is described in connection with Fig. 1 below.

It is known that if the transit time of the scatterers through the range cell is comparable to the inverse of the Doppler frequency, the Doppler output spectrum will be broadened accordingly. It is shown below that when the finite bandwidth of the transmitted signal is taken into account, the effects of transit time through the range cell are also taken care of automatically. When the transit time of the scatterers through the region from which coherent signals return to the Doppler system are limited by the *beam geometry*, however, different broadening effects occur in the Doppler output spectrum. These effects will be considered in a future communication. Thus, in this paper the phrase "transit time limited" will refer to situations where the beam geometry does not play a role.

The remainder of this paper starts by deriving a relation between the transmitted and output power spectral densities of random signal Doppler systems using broad bandwidth analysis and thus taking transit time effects into account. This expression, which is identical with that of Altes for pulsed RF

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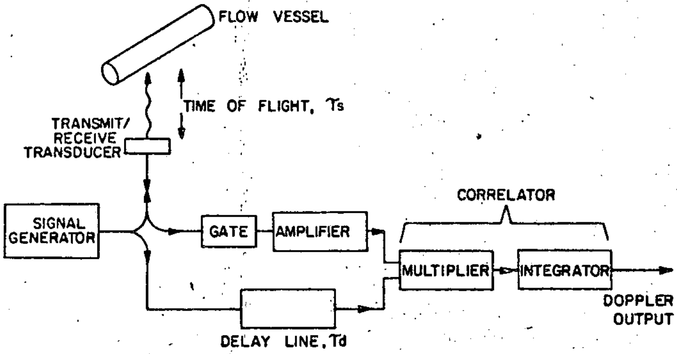


Fig. 1. Correlation system which can model random as well as deterministic Doppler systems.

Doppler systems, is then applied to several cases important in flow measurement, and some of the results are confirmed experimentally. In particular, it is shown that both expression (1) for high-resolution velocity estimates and expression (2) for velocity averaging must be multiplied by correction factors when the ratio of transmitted bandwidth to center frequency is finite. Since high-resolution measurements require large bandwidths, and since flow measurement at large ranges require the use of the lowest possible center frequency so as to minimize absorption, ratios of B/f_o close to unity will be required for transcutaneous high-resolution blood velocity measurements deep in the body, or for long range fluid flow measurements in general.

The Transit Time Limited Doppler Spectrum

A system is shown in Fig. 1 which can model all known ultrasonic Doppler flowmeters including random and pseudorandom types as well as deterministic continuous and pulsed RF types which mix the echo signal with a time gated reference signal. The signal generator can be assumed to transmit a signal which may be random or pseudorandom, pulsed RF or continuous RF. The transducer receives echoes from any stationary or moving scatterers situated in its ultrasound beam. A sample of the transmitted signal is passed through the delay line and correlated with the echoes received by the transducer. The center of the range cell is determined by the length of the delay line. Specifically, the time of flight from the transducer to the center of the range cell and back is equal to the time required for sound to traverse the delay line. The extent ΔR of this range cell is determined by the transmitted spectrum bandwidth B according to the relation

$$\Delta R \sim \frac{c}{2B} \tag{3}$$

where c is the sound velocity. This has been shown to hold for both deterministic and random signal Doppler systems [8].

We now derive the relation between the transmitted and output power spectral densities of random signal Doppler systems which may be stated as follows.

If a random stationary signal of time average power spectral density $S_x(\omega)$ illuminates a random stream of scatterers whose density is Poisson distributed, then the correlation of the transmitted signal and the echo has the spectral power density

$$S_z(\omega) = \overline{a^2} \int_{-\infty}^{\infty} \frac{\rho(\alpha)}{|\alpha|} \left| S_x\left(\frac{\omega}{\alpha}\right) \right|^2 d\alpha \tag{4}$$

provided that the scatterer transit time is limited by the properties of the transmitted signal and not by the beam geometry. Here $\alpha = 2v/c$ where v is the flow velocity component in the direction of the ultrasound beam and c is the velocity of sound. $\rho(\alpha)$ is the scatterer density in velocity space averaged over the range cell in the direction of the beam, and $\overline{a^2}$ is the attenuation factor of the signal reflected from each scatterer, averaged over the range cell. It is assumed that $\rho(\alpha)$ is a "slowly time varying" function; i.e., the relative position of the scatterers with regard to one another is assumed to remain substantially unchanged as they traverse the range cell of the Doppler system.

The proof of (3) starts from a rather complex relation of Chadwick's [11, eq. (2-13)] whose derivation is too lengthy to reproduce here. We will, however, attempt to provide motivation for this relation by the following heuristic argument.

Consider that the system of Fig. 1 transmits a signal $x(t)$. The reflected signal from one scatterer will then be $x[(1+\alpha)t - \tau]$ where τ is the echo delay at $t = 0$. The corresponding correlator output will be

$$z(t) = E\{x(t - \tau_r)\, x[(1+\alpha)t - \tau]\}$$

where τ_r is the delay of the reference signal and $E\{\ \}$ is the expectation of the transmitted signal averaged over the time constant of the correlator. This equation may be written as

$$z(t) \sim R_x[-\alpha t + \tau - \tau_r]$$

where $R_x[\tau]$ is the autocorrelation function of $x(t)$ and is approximately equal to $\langle x(t)x(t+\tau)\rangle$ averaged over the correlator time constant. It is known[1] that for a random ensemble of such scatterers of uniform velocity whose density is determined by a Poisson distribution, the correlator output will have an autocorrelation function given by

$$R_z(t_1, t_2) = \tfrac{1}{2} \rho V E\{z(t_1) z(t_2)\} \tag{5}$$

where ρ is the scatterer density and V is the volume of the system. Chadwick [11] has generalized (5) to the case of a stationary random signal illuminating a random Poisson distributed array of scatterers. In using his expression, we will ignore terms due to uncorrelated clutter echoes from outside the range cell and due to thermal receiver noise, since their inclusion would complicate the analysis and since their effects can be minimized by techniques unrelated to the subject matter of this paper. We will also ignore a sum frequency Doppler term which is filtered out in all practical systems. Within these restrictions Chadwick [11] finds that a random signal system illuminating a random array of moving scatterers produces a correlator output autocorrelation function very similar to (5) provided that the time expectation in that equation is replaced by an integration over velocity and real space. Namely,

$$R_z(t_1, t_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(\alpha, \tau) a^2(\tau) R_x(-\alpha t_1 + \tau - \tau_r) \cdot R_x(-\alpha t_2 + \tau - \tau_r)\, d\alpha\, d\tau.$$

Here $\rho(\alpha, \tau)$ is the time averaged scatterer density in velocity

[1] See, for instance, Brodie and Meindl [10].

space and "delay" space and $a(\tau)$ combines the effect of path attenuation with the time and spatially averaged scatterer cross section at a distance r from the transducer. It is assumed that the distance between the target and transducer is large enough that the plane wave approximation is valid. The range r is related to the delay coordinate by the expression

$$\tau = \frac{2r}{c}.$$

We will consider geometries where the velocity changes little across the range cell in the beam direction and where the transit time is not determined by the ultrasound beam width. An example is the case shown in Fig. 2(a) where the beam width is wider than the fluid carrying vessel diameter which is itself much larger than the length of the range cell. In such a case the above expression may be written as

$$R_z(t_1, t_2) = \overline{a^2} \int_{-\infty}^{\infty} \rho(\alpha) \left\{ \int_{-\infty}^{\infty} R_x[-\alpha t_1 + \tau - \tau_r] \cdot R_x[-\alpha t_2 + \tau - \tau_r] \, d\tau \right\} d\alpha.$$

Here $\rho(\alpha)$ is the density in velocity space, averaged across lines drawn across the range cell in the beam direction, and $\overline{a^2}$ is averaged across the whole volume of the range cell within the beam. Note that in the case illustrated in Fig. 2(a) all values of $\rho(\alpha)$ which exist in the blood vessel are encompassed by the range cell. Thus in this case the system of Fig. 2(a) should produce a doppler output spectrum containing components corresponding to all flow velocities present in the vessel.

We can obtain the spectral power density $S_z(\omega)$ of the correlator output by Fourier transforming $R_z(t_1, t_2)$ with respect to $t_1 - t_2$. Some algebra [12] shows that this gives

$$S_z(\omega) = \overline{a^2} \int_{-\infty}^{\infty} \frac{\rho(\alpha)}{|\alpha|} \left| S_x\left(\frac{\omega}{\alpha}\right)\right|^2 d\alpha \tag{4}$$

where $S_x(\omega)$ is the Fourier transform of $R_x(\tau)$ and is the spectral power density of the transmitted signal $x(t)$.

Since the expression has been derived for a stationary random signal, it should also hold for any periodic deterministic signal provided that $S_x(\omega)$ and $S_z(\omega)$ are now interpreted as energy spectral densities. This has in fact been proven by Altes [7] as mentioned above. Note that whereas Altes' result is restricted to scatterers which occupy a range of velocities at each point in space, our result is derived for incompressible liquids, where the scatterer velocity is a function of position. Furthermore, our method of derivation, which differs from Altes', brings out the dependence of the result on transit time effects, and can, therefore, readily be adapted to the important case where the transit time effects are determined by beam geometry [13], [14].

Application to Specific Cases

A. Flow Averaged Across a Vessel

This situation corresponds to the geometry of Fig. 2(a) where the ultrasound beam is wider than the fluid carrying vessel. If the scatterer denisty p is assumed uniform, and if the

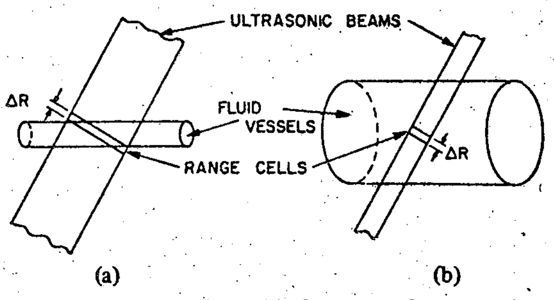


Fig. 2. Fluid flow geometries. (a) Geometry for measuring average flow. (b) Geometry for measuring point velocities.

velocity $\alpha(r)$ at a point is a function only of its distance r from the vessel axis, then it can be shown that the number of scatterers in the velocity range α to $\alpha + \Delta\alpha$ is

$$\rho(\alpha)\Delta\alpha = 2\pi r \sec\theta \, p \frac{dr}{d\alpha} \Delta\alpha. \tag{5}$$

Here $2\pi r \sec\theta$ is the circumference of the ellipse defined by the intercept of a cylinder of radius r with the center plane of the range cell. The cylinder is assumed concentric with the axis of the vessel.

Substituting (5) into (4), we find that

$$S_z(\omega) = \text{const} \int_0^R \frac{S_x^2[\omega/\alpha(r)]}{\alpha(r)} r\,dr \tag{6}$$

where R is the radius of the vessel.

The fluid velocity profile can be written in terms of α as

$$\alpha(r) = \alpha_m \left\{1 - \left(\frac{r}{R}\right)^n\right\} \tag{7}$$

where α_m corresponds to the maximum fluid velocity along the center of the vessel. For steady flow of a Newtonian liquid, $n = 2$ in (7). Higher values of n correspond to blunter velocity profiles, as shown in the insert of Fig. 3. Note that $n = \infty$ corresponds to uniform velocity throughout the vessel.

Doppler output spectra $S_z(\omega)$ for a series of these velocity profiles are computed in Fig. 3 for the case of a truncated bell-shaped transmitted spectrum $S_x(\omega)$. This figure shows that if $S_x(\omega)$ is known, the velocity profile and α_m can be identified from the Doppler output spectrum $S_z(\omega)$, provided that $\alpha(r)$ is one of a known family of centrosymmetric functions. Once this has been established, the flow parameter α_m can be derived from the upper cutoff frequencies, thus allowing the mean flow to be calculated.

A far more elegant way of estimating average flow is that of forming the normalized first moment of the output spectrum as shown in (2). This relation was derived while neglecting the bandwidth of the transmitted signal [9], [10]. We will find that a modified form of this relation holds even when transit time effects are taken into account.

We will restrict the analysis to flow in a tube, all of which is in one direction, since in the system being analyzed negative flows cannot be distinguished from positive flows. Under these circumstances, (4) can be written as

$$S_z(\omega) = \overline{a^2} \int_0^{\infty} \frac{\rho(\alpha)}{\alpha} \left| S_x\left(\frac{\omega}{\alpha}\right)\right|^2 d\alpha.$$

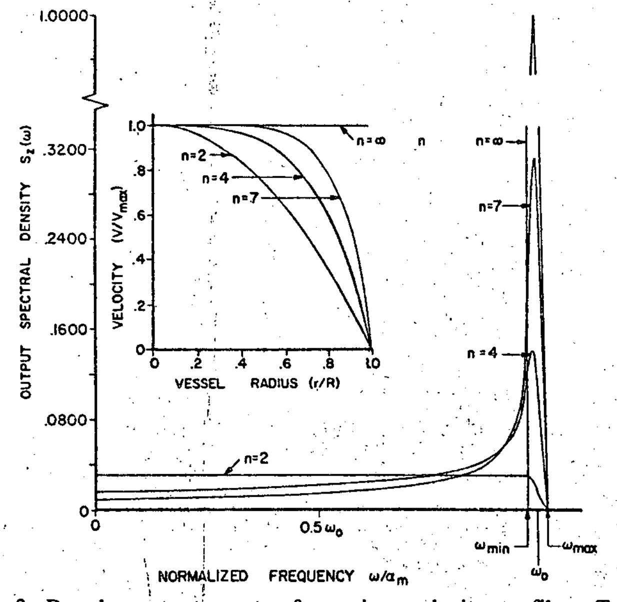


Fig. 3. Doppler output spectra for various velocity profiles. Transmitted spectrum:

$$S_x(\omega) = \frac{(\pi B)^2}{(\pi B)^2 + (\omega - \omega_o)^2}$$

truncated at $\omega = \omega_o \pm B\pi$. Insert: Velocity versus distance from cylinder axis for various values of n in (7).

Using the substitution $y = \omega/\alpha$ and inverting the order of integration, we then find that

$$\frac{\int_0^\infty \omega S_z(\omega)\, d\omega}{\int_0^\infty S_z(\omega)\, d\omega} = \frac{\int_0^\infty \alpha\rho(\alpha)\, d\alpha}{\int_0^\infty \rho(\alpha)\, d\alpha} \frac{\int_0^\infty y S_x^2(y)\, dy}{\int_0^\infty S_x^2(y)\, dy}, \tag{8}$$

i.e.,

$$\frac{\int_0^\infty \omega S_z(\omega)\, d\omega}{\int_0^\infty S_z(\omega)\, d\omega} = \bar{\alpha} \frac{\int_0^\infty y S_x^2(y)\, dy}{\int_0^\infty S_x^2(y)\, dy}. \tag{9}$$

It is interesting to note that this equation is independent of the form of the velocity profile $\rho(\alpha)$. Inspection shows that the rhs of (9) reduces to $\bar{\alpha}\omega_o$ which is equivalent to (2), where ω_o is the center frequency of the transmitted single-sided spectrum, whenever that spectrum is symmetrical about some center frequency. Then, under these circumstances, it is justified to neglect the finite bandwidth of the transmitted signal. However, the single-sided spectrum $S_x(\omega)$ usually does not have an axis of symmetry, particularly when broad-band transducers are used, or when this spectrum is designed to cope with Rayleigh scattering and frequency dependent absorption effects. Under these circumstances the more general (9) must be used to estimate average flow velocity.

A number of other results which can be proved analytically [12] are also illustrated by Fig. 3.

1) For a given $S_x(\omega)$ the area under each curve is the same. This is a consequence of the fact that the total energy reflected from a fluid-carrying vessel cannot depend on the form of the velocity profile.

2) For the parabolic velocity profile, ($n = 2$), $S_z(\omega)$ is constant up to the frequency $\alpha_m\omega_{min}$ where ω_{min} is the lower cutoff frequency of the transmitted spectrum. This is a consequence of the fact, provable from (5) and (8), that the density in velocity space, $\rho(\alpha)$, is not a function of α for $n = 2$.

3) The upper cutoff frequency for the Doppler output spectra is $\alpha_m\omega_{max}$ where ω_{max} is the upper cutoff frequency of the transmitted spectrum.

B. Spectrum for Uniform Flow Velocity

Conditions under which an ultrasound beam intercepts fluid moving at uniform velocity, whose transit time is limited by the transmitted signal, are as follows.

1) The beam is much wider than the fluid carrying vessel.

2) The beam is crossed by a jet of fluid.

3) The beam is much narrower than the vessel, as shown in Fig. 2(b), but is angled in such a way that the transit time of most of the fluid crossing it is determined by the range cell rather than the beam width.

In all these cases α is a constant in (4) so that the Doppler output spectrum becomes

$$S_z(\omega) = \text{const} \left| S_x\left(\frac{\omega}{\alpha_m}\right)\right|^2. \tag{10}$$

This uniform fluid velocity case corresponds to the curve $n = \infty$ in Fig. 3. This illustrates the result which follows from (10), that for uniform velocity measurements the Doppler system output spectrum peaks at $\alpha_m\omega_o$ where ω_o is the peak energy frequency of the transmitted spectrum. It can also be seen that the Doppler output spectrum has a bandwidth of order $\alpha_m B$ where B is the bandwidth of the transmitted spectrum. This result is a consequence of the fact that the fluid transit time through the range cell given by (3) is of order

$$\Delta t = \frac{1}{v}\frac{c}{2B} \sim \frac{1}{\alpha B}.$$

C. Velocity Gradient Correction Factor

When measuring velocity profiles in small vessels, transit time effects become important, particularly near the vessel edges where the velocity gradient approaches infinity. It is, therefore, pertinent to calculate the errors made in estimating velocity profiles when transit time effects are ignored. This can readily be done for the geometry of Fig. 2(b) where the range cell is much smaller than the vessel diameter, but where the beam width is not.

If the velocity parameter varies over the range $\alpha_1 - \alpha_2 = \Delta\alpha$ over the width of the ultrasound beam, then with transit time effects neglected, the peak of the Doppler output spectrum $S_z(\omega)$ would be calculated to occur at the frequency

$$\omega_p = \tfrac{1}{2}(\alpha_1 + \alpha_2)\omega_o = \bar{\alpha}\omega_o$$

where ω_o is the frequency of the peak energy of the transmitted spectrum, and $\bar{\alpha}$ is the mean of α_1 and α_2.

To calculate the frequency corresponding to the peak energy of the Doppler output spectrum $S_z(\omega)$, we start from (4) and assume conditions of steady flow with a parabolic velocity profile so that ρ is not a function of α. Then from (4)

$$S_z(\omega) = \text{const} \int_{\alpha_1}^{\alpha_2} \frac{1}{|\alpha|} S_x^2\left(\frac{\omega}{\alpha}\right) d\alpha. \quad (11)$$

The peak of $S_z(\omega)$ will occur for that frequency for which $\partial S_z(\omega)/\partial\omega = 0$. Differentiating both sides of (11) with regard to ω and changing variables to $y = \omega/\alpha$ gives

$$\frac{\partial S_z(\omega)}{\partial \omega} = \frac{2}{\omega} \text{const} \int_{\omega/\alpha_2}^{\omega/\alpha_1} S_x(y) \frac{\partial S_x(y)}{\partial y} dy. \quad (12)$$

If the transmitted spectrum $S_x(\omega)$ is symmetrical about ω_o, then $\partial S_x(\omega)/\partial\omega$ will be an antisymmetrical function passing through zero at ω_o. In view of this property (12) indicates that $\partial S_z(\omega)/\partial\omega$ will be zero when the limits of integration are symmetrical with respect to ω_o, i.e., when $\omega_o - \omega/\alpha_2 = \omega/\alpha_1 - \omega_o$. Thus the peak frequency of the Doppler spectrum, taking transit time effects into account, will be

$$\omega_p = \frac{2\omega_o}{\frac{1}{\alpha_1} + \frac{1}{\alpha_2}} = \bar{\alpha}\left\{1 - \left(\frac{\Delta\alpha}{2\bar{\alpha}}\right)^2\right\}\omega_o. \quad (13)$$

This relation shows that for parabolic velocity profiles, neglect of transit time effects causes a velocity overestimate of 100 $(\Delta v/2v)^2$ percent in high-resolution flow estimates taken in a region of average velocity v in which there exists a range of velocities Δv.

Since for high-resolution flow measurement the Doppler output spectrum has a width of the order of αB around a center frequency αf_o, a range of velocities $\Delta\alpha$ will only be unresolved when $f_o \Delta\alpha$ is less than a quantity of the order of αB, i.e., when

$$\frac{\Delta\alpha}{\alpha} = \frac{\Delta v}{v} \leqslant \frac{B}{f_o}.$$

Thus the maximum fractional correction which is required in regions of high-velocity gradient is of order $(B/2f_o)^2$. This approaches 25 percent as the ratio of transmitted bandwidth to center frequency approaches unity.

Experimental Verification

The prediction that random transmitted signals produce the same Doppler output spectra as pulsed RF signals having the same spectrum has already been verified by us experimentally [8]. We present here experimental verification of two of the specific spectra calculated above. These are for the case of steady parabolic flow where the beam is much wider than the vessel so that flow is averaged across the entire vessel and the case of a single velocity stream. The predicted output spectra for both these cases are illustrated in Fig. 3.

Fig. 4(a) shows an experimental Doppler output spectrum for steady Newtonian flow averaged across a 3.3-mm internal diameter vessel with an ultrasound beam diameter of 12 mm. A cellulose fiber suspension was used as the flow medium.[2] The fibers are 2–20 μ long, which is sufficiently small to assure uniform distribution of scatterers thoughout the diameter of the flow vessel. The spectrum can be seen to be flat as the theory predicts and decreases to zero in the expected manner. For this measurement the transducer was held at 45° to the flow stream and transmitted 5-μs bursts of RF centered at 5 MHz (giving $\omega_{max} - \omega_{min} = 2.5 \times 10^6\ s^{-1}$). The spectrum can be seen to fall to zero in slightly less than one half of a major division, or in this case slightly less than 200 Hz. For the experimental conditions described, the theory predicts that the spectrum should fall to zero in 170 Hz. (The decreasing spectral amplitude at the lower Doppler frequencies can be accounted for to within the experimental uncertainties by the bandpass characteristics of the filter used to extract the Doppler signal from the return wall echoes. The *RC* time constant of this filter high-pass section corresponds to a 3-dB point of 200 Hz, as the figure shows.)

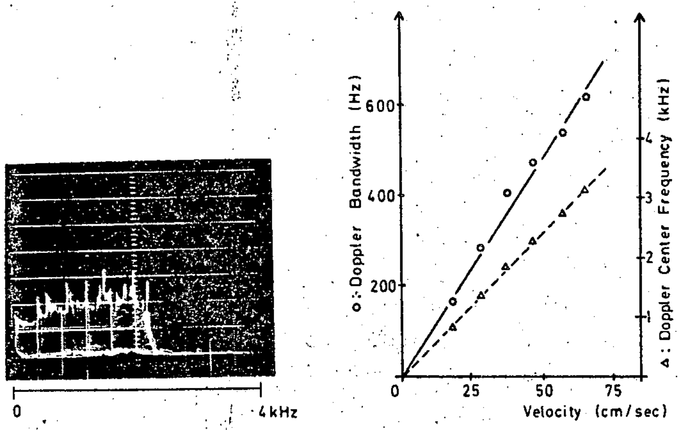
To simulate single or point velocity measurements, an apparatus was constructed to drive a continuous loop thread at constant speed. With the thread moving through a broad ultrasound beam and the range cell held fixed, the prediction that the Doppler spectrum is a compressed version of the transmitted spectrum, related by the factor α, was verified [see Fig. 4(b)] by demonstrating that the bandwidth and center frequency of the Doppler spectrum are both proportional to the thread velocity. Although RF was used in the experiments illustrated here, similar results would be expected using noise as the transmitted signal.

Discussion

The first theoretical conclusion of this paper states that pulsed RF and random signal Dopplers produce output spectra having identical envelopes providing that the same holds true for their transmitted spectra. This may be considered to have been proved experimentally in our earlier work [8]. It is pertinent to examine to what extent the experimental results presented in the present paper validate (4). The fact that the Doppler spectrum shown in Fig. 4(a) for flow averaged across a vessel is flat, is merely a reflection of the fact that the density spectrum of Newtonian flow in a cylinder is uniform in velocity space. However, the rough agreement between the predicted and measured slope of the upper frequency cutoff suggests that (4) is indeed of the right form. This agreement also indicates that the assumption that $\rho(\alpha)$ is slowly varying, is valid for the particular fluid flow situation examined here. This particular assumption is not tested by the moving thread experiment [see Fig. 4(b)], since the thread irregularities which reflect the ultrasound will obviously always stay fixed with regard to one another. However, the fact that the Doppler spectrum bandwidth for this case is found to be proportional to its center frequency as predicted from (10) does indicate that this relation is at least of the right form.

It will be shown in a future communication that the analytical techniques used here can be used to calculate the effects of Rayleigh scattering and frequency dependent absorption on the Doppler spectrum. Frequency dependent absorption effects were unimportant in the experiments reported here because the transducers and test vessels were immersed in water which has negligibly small absorption compared to the loss caused by scattering. Rayleigh scattering effects will be shown to sharpen the cutoff of the $n = 2$ spectrum and to

[2] Brinkman Instruments MN300 cellulose fibers.



(a) (b)

Fig. 4. (a) Doppler spectrum measured for steady flow averaged over the entire vessel with transit time limited by the range cell. (b) Doppler output spectrum bandwidth and center frequency versus thread velocity with transit time limited by the range cell.

shift the $n = \infty$ spectrum of Fig. 3. These effects are found to be proportional to $(B/f_o)^2$ and are not strong enough to be detected under the present experimental conditions.

Conclusions

The findings of the current paper may be summarized as follows. An expression has been derived which relates the Doppler output spectrum of ultrasonic flow measurement systems to the transmitted spectrum for stationary random transmitted signals. The result agrees with an earlier one devised for deterministic transmitted signals. This proves that the Doppler output spectrum envelope depends only on the spectrum envelope of the transmitted signal and is independent of whether this signal is deterministic or stochastic. Our expression for the output spectrum (4) takes transit time effects rigorously into account, provided that these are due to the properties of the transmitted signal, rather than to the geometry of the ultrasound beam. The result has been used to calculate the Doppler output spectrum for a series of velocity profiles assuming broad bandwidth transmitted spectra which would be used in high-resolution fluid velocity measurements. The results demonstrate that if the fluid velocity profile is known or is one of a known family of curves, then the Doppler output profile can be used to estimate its parameters provided the transit time is determined by the signal range cell rather than the beam diameter. It is also shown (2) that for estimating average velocity using the first moment of the output spectrum is valid even for a finite bandwidth transmitted spectrum provided that the spectrum is symmetric about a center frequency. A correction factor for assymetrical spectra is given in (8).

It has also been established that for high-resolution velocity estimates taken in regions of high-velocity gradient, the velocity estimate using the conventional Doppler (1) is too large by $+100\ (\Delta v/2v)^2$ percent where Δv is the range of velocities encompassed by the width of the ultrasound beam, and v is the velocity in the center of the range cell. This type of correction cannot exceed $100\ (B_o/2f_o)^2$ percent. It will be shown in a future communication that Rayleigh scattering effects cause correction factors of the same order of magnitude which are however not dependent on velocity gradient.

Finally, it is noteworthy that although the quantity $\alpha = 2v/c$ can be estimated, v being the component of velocity in the direction of the beam, the actual fluid velocity and the angle between it and the ultrasound beam cannot be estimated separately from the properties of the Doppler output spectra discussed here. It has been predicted, however [14], that these two quantities can be estimated independently under conditions where the transit time is limited by the beam geometry rather than the range cell. Preliminary experimental tests of this hypothesis [15] show that this prediction is valid. However it is found that beam divergence, which becomes more important the closer the target is to the transducer, can substantially broaden the spectrum over the value calculated for the case of the plane wave approximation. It must be remembered that the plane wave approximation, which has been used for the calculations presented in this paper, is only valid when the target is very far from the transducer or when the transducer beam has been collimated by means of lenses. When this is not the case, the spectra obtained may be broader than the ones calculated in this paper.

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