Comparison of the average specific absorption rate in the ellipsoidal conductor and dielectric models of humans and monkeys at radio frequencies

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Perturbation theory has been used to find the first-order internal electric field, the SAR (specific absorption rate), the spatial variations of the SAR, and the maximum SAR in prolate spheroidal and ellipsoidal models of man and experimental animals during irradiation by an electromagnetic plane wave when the wavelength is long as compared to the dimensions of the exposed body. In our "conductor" model of man, conductivity is written explicitly in the curl H equation as: $\nabla \times \mathbf{H} = \sigma\mathbf{E} + j\omega\epsilon\,\mathbf{E}$. In what we call the "dielectric" model, the conductivity is contained implicitly in the complex permittivity, so that the curl H equation is $\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E}$. The two models give different results for first-order fields because the equations are expanded in a power series in k ($k = \omega\sqrt{\mu\epsilon}$); in the conductor model σ enters into the zero-order equations but in the dielectric model it does not. Because of the nature of the zero-order equations, the expressions obtained from the conductor model are not valid as $\sigma \to 0$. We have found that the conductor model is valid only if $\epsilon_2 \gg \epsilon_1$ where ϵ_1 and ϵ_2, respectively, are the real and imaginary parts of the complex dielectric constant of the models. Consequently, some caution must be exercised in applying the results of perturbation theory as based on the conductor model. In this paper, the results of perturbation theory as applied to a lossy dielectric ellipsoidal model are described. The SAR in a dielectric ellipsoidal model of a rhesus monkey is calculated and compared with that of the conductor model. The SAR in the two models is found to be the same if the conduction current in the body is much larger than the displacement current. Although the conductor model is inaccurate for low values of conductivity, the equations are simpler than the ones for the dielectric model, and hence the conductor model is advantageous when valid.

1. INTRODUCTION

The perturbation theory described by *Van Bladel* [1964] has been used to find the first-order internal electric field, the power distribution, and the SAR (specific absorption rate in W/kg) in prolate spheroidal and ellipsoidal conductor models of man and experimental animals during irradiation by an electromagnetic plane wave when the wavelength is long compared to the dimensions of the body [*Durney et al.*, 1975; *Johnson et al.*, 1975; *Massoudi et al.*, 1977a,b].

Expressions for the first-order internal fields and for the SAR in the conductor models are valid only if $\epsilon_2 \gg \epsilon_1$, ϵ_1 and ϵ_2, respectively, being the real and imaginary parts of the complex dielectric constant of the models. In order to remove the above restriction, we applied perturbation theory to a lossy dielectric ellipsoidal model to obtain expressions for the first-order internal fields and for the SAR, retaining both the real and imaginary parts of the complex dielectric constant, $\epsilon = \epsilon_1 - j\epsilon_2 \equiv \epsilon_1 - j\sigma/\omega\epsilon_0$. We shall refer to this model as a dielectric model and to the ones reported by *Durney et al.* [1975], *Johnson et al.* [1975], and *Massoudi et al.* [1977a,b] as conductor models.

We have compared results obtained for the conductor and for the dielectric ellipsoidal models of a rhesus monkey. The results of the average SAR calculations for the prolate spheroidal model of man are compared with those of the extended boundary condition method (EBCM) [*Barber*, 1976]. Since the derivation of the internal and scattered fields for the dielectric model is different from that of the conductor model, a brief description of perturbation theory and the method of derivation of the internal fields for one polarization are given in the next section.

2. DESCRIPTION OF THE THEORY

As described by *Durney et al.* [1975], the set of fields, interior, incident, and scattered, are expanded in a power series in $(-jk)$, k being the free-space propagation constant.

$$\mathbf{E} = \sum_{n=0}^{\infty} \mathbf{E}_n (-jk)^n$$

$$\mathbf{E}^i = \sum_{n=0}^{\infty} \mathbf{E}_n^i (-jk)^n$$

$$\mathbf{E}^s = \sum_{n=0}^{\infty} \mathbf{E}_n^s (-jk)^n$$

where $\mathbf{E}$, $\mathbf{E}^i$, and $\mathbf{E}^s$ are, respectively, the interior, incident, and scattered fields. Similar expansions are developed for the magnetic fields. Relationships among the expansion coefficients can be obtained by substituting the power series for electric and magnetic fields in Maxwell's equations and equating like powers of $(-jk)$. Thus, for the interior fields,

$$\nabla \times \mathbf{E}_0 = 0 \tag{1}$$

$$\nabla \times \mathbf{E}_n = \eta_0 \mathbf{H}_{n-1} \qquad n \geq 1 \tag{2}$$

$$\nabla \times \mathbf{H}_0 = 0 \tag{3}$$

$$\nabla \times \mathbf{H}_n = -(\epsilon/\eta_0)\mathbf{E}_{n-1} \qquad n \geq 1 \tag{4}$$

$$\nabla \cdot \mathbf{E}_n = 0 \tag{5}$$

$$\nabla \cdot \mathbf{H}_n = 0 \tag{6}$$

and the scattered fields satisfy the following set of equations:

$$\nabla \times \mathbf{E}_0^s = 0 \tag{7}$$

$$\nabla \times \mathbf{E}_n^s = \eta_0 \mathbf{H}_{n-1}^s \qquad n \geq 1 \tag{8}$$

$$\nabla \times \mathbf{H}_0^s = 0 \tag{9}$$

$$\nabla \times \mathbf{H}_n^s = -(1/\eta_0)\mathbf{E}_{n-1}^s \qquad n \geq 1 \tag{10}$$

$$\nabla \cdot \mathbf{E}_n^s = 0 \tag{11}$$

$$\nabla \cdot \mathbf{H}_n^s = 0 \tag{12}$$

where

$$\epsilon = \epsilon_1 - j\epsilon_2$$

$$\epsilon_2 = \sigma/\omega\epsilon_0$$

$$\eta_0 = \sqrt{\mu_0/\epsilon_0}$$

ϵ_1 and ϵ_2, respectively, are the real and the imaginary part of the complex permittivity. The conductivity of the medium is σ, and $e^{j\omega t}$ time variations are assumed. The interior and the exterior fields are coupled by the following boundary conditions at the surface of the scatterer:

$$\hat{\mathbf{n}} \times \mathbf{E}_n = \hat{\mathbf{n}} \times (\mathbf{E}_n^i + \mathbf{E}_n^s) \tag{13}$$

$$\hat{\mathbf{n}} \times \mathbf{H}_n = \hat{\mathbf{n}} \times (\mathbf{H}_n^i + \mathbf{H}_n^s) \tag{14}$$

$$\hat{\mathbf{n}} \cdot (\epsilon\mathbf{E}_n) = \hat{\mathbf{n}} \cdot (\mathbf{E}_n^i + \mathbf{E}_n^s) \tag{15}$$

$$\hat{\mathbf{n}} \cdot \mathbf{H}_n = \hat{\mathbf{n}} \cdot (\mathbf{H}_n^i + \mathbf{H}_n^s) \tag{16}$$

where $\hat{\mathbf{n}}$ is the outer unit normal vector at the boundary. The scatterer is assumed to be nonmagnetic. It should be pointed out that the difference between the conductor and the dielectric models comes from the way the complex dielectric constant is treated in the expansions of the equation for the interior magnetic fields (equations (3) and (4)) and the boundary conditions for $\mathbf{E}_n$ (equation (15)). In the conductor model, the complex dielectric constant is first written as

$$\epsilon = \epsilon_1 - (j\sigma/\epsilon_0\omega) = \epsilon_1 - \sigma\eta_0(-jk)^{-1}$$

and then the fields are expanded in powers of $(-jk)$ whereas in the dielectric model ϵ is considered as $\epsilon = \epsilon_1 - j\epsilon_2$. The appearance of the term $(-jk)^{-1}$ in ϵ for the conductor model causes the difference between the relations of the nth-order fields in the two models. In the following sections, it will be shown that the results obtained with the conductor model are valid only if the displacement current in the body is much smaller than the conduction current, or $\epsilon_1 << \epsilon_2$, whereas the dielectric model does not have this restriction. In the dielectric model, on the other hand, the frequency dependence of ϵ_2 is not explicitly taken into account in the power-series expansion. The extent of possible errors introduced by neglecting the frequency dependence of ϵ_2 in the expansion has not been determined for the general case but for specific cases comparison with results obtained from the EBC method [*Barber*, 1976] indicate that the error is small.

3. FIRST-ORDER INTERNAL FIELDS FOR THE DIELECTRIC ELLIPSOID IRRADIATED BY AN ELECTROMAGNETIC PLANE WAVE

In this section, the solution of the zeroth and the first-order internal electric fields for a plane wave incident on a dielectric ellipsoid is given. The coordinate system with respect to the ellipsoid is oriented as shown in Figure 1.

The equation of the ellipsoid in the coordinate system of Figure 1 is:

$$(x^2/a^2) + (y^2/b^2) + (z^2/c^2) = 1 \tag{17}$$

where a, b, and c are the semiprincipal axes of the ellipsoid with $a > b > c$.

Expressions for the internal electric fields and for the SAR are given for each of six primary polarizations. The polarization is defined by the relation of the vectors $\mathbf{E}^i$, $\mathbf{H}^i$, $\mathbf{K}$ with respect to axes a, b, c. Thus *EKH* polarization is defined as the orientation for which $\mathbf{E}^i$ lies along a, $\mathbf{H}^i$ lies along **b**, and **K** lies along **c**, where the length of each axis has been used to designate the axis. The six polarizations are listed in Table 1.

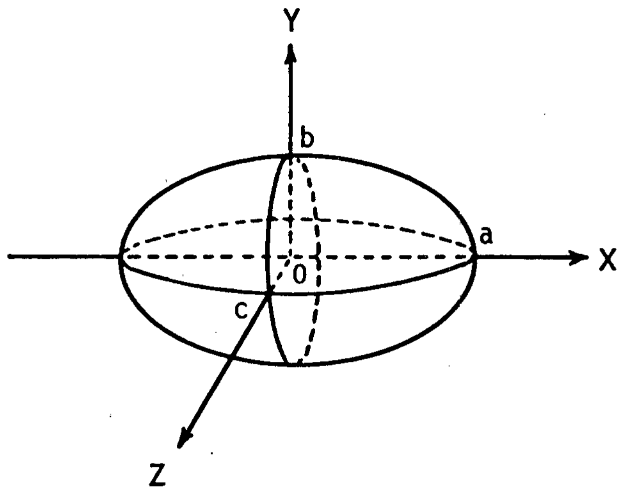

Fig. 1. Orientation of the coordinate system with respect to the ellipsoid.

3.1. *Derivation for EKH polarization.*

For *EKH* polarization the incident fields have the following forms:

$$\mathbf{E}^i = e^{-jky}\,\hat{\mathbf{x}} = [\sum_{n=0}^{\infty} (-jky)^n/n!]\,\hat{\mathbf{x}} \tag{18}$$

$$\mathbf{H}^i = -(1/\eta_0)e^{-jky}\hat{\mathbf{z}} = -(1/\eta_0)[\sum_{n=0}^{\infty} (-jky)^n/n!]\,\hat{\mathbf{z}} \tag{19}$$

3.1.1. *Zeroth-order fields.* Equations (1), (5), (7), and (11) require that all the zeroth-order fields must have zero divergence and zero curl. These fields can be expressed as the gradient of scalar potentials, that is, $\mathbf{E}_0 = \nabla\Phi_0$, $\mathbf{E}_0{}^s = \nabla\Phi_0{}^s$, and $\mathbf{E}_0{}^i = \nabla\Phi_0{}^i$. These scalar potentials must satisfy Laplace's equation, since $\nabla \cdot \mathbf{E}_0 = 0$. In ellipsoidal coordinates Laplace's equation has the following form [*Stratton,* 1941]:

TABLE 1. Definitions of polarization for ellipsoids.

Polarization	Vector Parallel to a, b, or c: a	b	c
EKH	E^i	K	H^i
EHK	E^i	H^i	K
KEH	K	E^i	H^i
KHE	K	H^i	E^i
HEK	H^i	E^i	K
HKE	H^i	K	E^i

$$(\eta-\zeta)R_\xi(\partial/\partial\xi)[R_\xi(\partial\Phi/\partial\xi)] + (\zeta-\xi)R_\eta(\partial/\partial\eta) \cdot[R_\eta(\partial\Phi/\partial\eta)] + (\xi-\eta)R_\zeta(\partial/\partial\zeta)[R_\zeta(\partial\Phi/\partial\zeta)] = 0 \tag{20}$$

with

$$R_s = [(s+a^2)(s+b^2)(s+c^2)]^{1/2} \qquad (s=\xi,\eta,\zeta) \tag{21}$$

The properties of the ellipsoidal harmonics that satisfy Laplace's equation, equation (20), can be found in the literature [*Whittaker and Watson,* 1946; *Morse and Feshbach,* 1953]. The solution for Φ_0, the internal potential, is given by *Stratton* [1941] as:

$$\Phi_0 = A_1\,x \tag{22}$$

Therefore,

$$\mathbf{E}_0 = \nabla\Phi_0 = A_1\,\hat{\mathbf{x}} \tag{23}$$

where

$$A_1 = [(abc/2)(\epsilon-1)I_a + 1]^{-1} \tag{24}$$

with

$$I_a = \int_0^\infty [(\xi+a^2)R_\xi]^{-1}d\xi \qquad a = a,b,c \tag{25}$$

3.1.2. *First-order fields.* The first-order electric fields inside the ellipsoid must satisfy equations (2) and (5), i.e., $\nabla \cdot \mathbf{E}_1 = 0$ and $\nabla \times \mathbf{E}_1 = \eta_0\mathbf{H}_0$. Since the ellipsoid is nonmagnetic, $\mathbf{H}_0 = -\hat{\mathbf{z}}/\eta_0$. Thus,

$$\nabla \times \mathbf{E}_1 = -\hat{\mathbf{z}} \tag{26}$$

We set

$$\mathbf{E}_1 = \mathbf{F}_1 + \nabla\Phi_1 \tag{27}$$

According to equation (5),

$$\nabla \cdot \mathbf{F}_1 = 0 \tag{28}$$

and

$$\nabla^2\Phi_1 = 0 \tag{29}$$

A solution of equation (28), by inspection, is

$$\mathbf{F}_1 = (y\hat{\mathbf{x}} - x\hat{\mathbf{y}})/2 \tag{30}$$

$\mathbf{E}_1{}^s$ must satisfy equations (8) and (11), i.e., $\nabla \cdot \mathbf{E}_1{}^s = 0$ and $\nabla \times \mathbf{E}_1{}^s = \eta_0\mathbf{H}_0{}^s$. But $\mathbf{H}_0{}^s = 0$; therefore,

$$\nabla \times \mathbf{E}_1{}^s = 0 \tag{31}$$

$\mathbf{E}_1{}^s$ can be written as the gradient of a scalar potential, as $\mathbf{E}_1{}^s = \nabla\Phi_1{}^s$, where $\Phi_1{}^s$ must satisfy Laplace's equation, since $\nabla \cdot \mathbf{E}_1{}^s = 0$.

The first-order incident electric field, from equation (18) can be written as

$$\mathbf{E}_1{}^i = y\hat{\mathbf{x}} \tag{32}$$

Substitution of equations (27) and (32) into equations (29) and (31) gives

$$\hat{\xi} \times (\nabla\Phi_1 - \nabla\Phi_1{}^s) = \hat{\xi} \times \nabla\psi \quad \text{at } \xi = 0 \quad (33)$$

and

$$\hat{\xi} \cdot (\epsilon\nabla\Phi_1 - \nabla\Phi_1{}^s) = \hat{\xi} \cdot (\mathbf{E}_1{}^i - \epsilon\mathbf{F}_1) \quad \text{at } \xi = 0 \quad (34)$$

with

$$\psi = xy/2 = (1/2)\{[(\xi + a^2)(\xi + b^2)$$

$$\cdot(\eta + a^2)(\eta + b^2)(\zeta + a^2)(\zeta + b^2)] / [(b^2 - a^2) \quad (35)$$

$$\cdot(c^2 - a^2)(c^2 - b^2)(a^2 - b^2)]\}^{1/2}$$

Equation (33) indicates that the potentials Φ_1 and $\Phi_1{}^s$ must have the same η and ζ dependence as ψ. We presume, therefore, that Φ_1 and $\Phi_1{}^s$ are functions of the form:

$$\Phi = Ag(\xi) f_1(\eta) f_2(\zeta) \quad (36)$$

with

$$f_1(\eta) = [(\eta + a^2)(\eta + b^2)]^{1/2} \quad (37)$$

$$f_2(\zeta) = [(\zeta + a^2)(\zeta + b^2)]^{1/2} \quad (38)$$

Substitution of equation (36) into the Laplace's equation, equation (20), gives:

$$R_\xi(d/d\xi)\{R_\xi[dg(\xi)/d\xi]\} - [(3/2)\xi + c^2 + (a^2 + b^2)/4] \quad (39)$$

$$\cdot g(\xi) = 0$$

One solution of equation (39) is:

$$g_1(\xi) = [(\xi + a^2)(\xi + b^2)]^{1/2} \quad (40)$$

Another independent solution is:

$$g_2(\xi) = g_1(\xi) \int [g_1{}^2(\xi) R_\xi]^{-1} d\xi \quad (41)$$

Only $g_1(\xi)$ is an admissible solution for the internal potential Φ_1 because $g_2(\xi)$ is infinite at $\xi = -c^2$, whereas $g_1(\xi)$ is finite at all points within the surface $\xi = 0$. But $g_2(\xi)$ will vanish properly at infinity if the upper limit of integration in equation (41) is made infinite [*Stratton*, 1941]. Therefore, the internal and the external potentials can be written as:

$$\Phi_1 = Bg_1(\xi) f_1(\eta) f_2(\zeta) \equiv B_c xy \quad (42)$$

$$\Phi_1{}^s = B_c{}^s g_2(\xi) f_1(\eta) f_2(\zeta) \quad (43)$$

The constants B_c and $B_c{}^s$ are determined from the boundary conditions, equations (33) and (34). The final results are:

$$B_c = \frac{2 + (\epsilon - 1)(a^2 - b^2)\, abc\, I_{ab}}{4 + 2(\epsilon - 1)(a^2 + b^2)\, abc\, I_{ab}} \quad (44)$$

$$B_c{}^s = \frac{ab^3 c(1 - \epsilon) A_0}{2 + (\epsilon - 1)(a^2 + b^2)\, abc\, I_{ab}} \quad (45)$$

with

$$A_0 = [(b^2 - a^2)(c^2 - a^2)(c^2 - b^2)(a^2 - b^2)]^{-1/2} \quad (46)$$

and

$$I_{ab} = (I_a - I_b)/(b^2 - a^2) \quad (47)$$

The first-order electric field inside the ellipsoid can be written as:

$$\mathbf{E}_1 = \mathbf{F}_1 + \nabla\Phi_1 = B_3\, y\hat{\mathbf{x}} + C_3\, x\hat{\mathbf{y}} \quad (48)$$

where

$$B_3 = 1/2 + B_c \quad (49)$$

$$C_3 = B_c - 1/2 \quad (50)$$

The electric field inside the ellipsoid to first order, according to equations (23) and (48) and the power series expansion for $\mathbf{E}$, is:

$$\mathbf{E} = \mathbf{E}_0 - jk\mathbf{E}_1 = (A_1 - jk\, B_3\, y)\hat{\mathbf{x}} - jk\, C_3\, x\hat{\mathbf{y}} \quad (51)$$

The above expression will be used for calculations of the SAR in the next section.

Expressions for internal electric fields to the first order for the *EHK*, *KEH*, *KHE*, *HEK*, and *HKE* polarizations are derived by following the same procedure as described above for *EKH* polarization. The final results for each of these polarizations are given below.

3.2. *Results for other polarizations.*

3.2.1. *EHK polarization.* The incident fields for this polarization are chosen to be $\mathbf{E}^i \| \hat{\mathbf{x}}, \mathbf{H}^i \| \hat{\mathbf{y}}$.

The internal fields are:

$$\mathbf{E}_0 = A_1\, \hat{\mathbf{x}} \quad (52)$$

$$\mathbf{E}_1 = C_2\, z\hat{\mathbf{x}} + B_2\, x\hat{\mathbf{z}} \quad (53)$$

$$\mathbf{E} = \mathbf{E}_0 - jk\mathbf{E}_1 = (A_1 - jkC_2\, z)\,\hat{\mathbf{x}} - jkB_2\, x\hat{\mathbf{z}} \quad (54)$$

where

$$C_2 = 1/2 - B_b \quad (55)$$

$$B_2 = -1/2 - B_b \quad (56)$$

$$B_b = -\frac{2 + (\epsilon - 1)(a^2 - c^2)\, abc\, I_{ca}}{4 + 2(\epsilon - 1)(a^2 + c^2)\, abc\, I_{ca}} \quad (57)$$

$$I_{ca} = (I_c - I_a)/(a^2 - c^2) \quad (58)$$

A_1 and I_a are given by equations (24) and (25), respectively.

3.2.2. *KEH polarization.* The incident fields are $\mathbf{E}^i \| \hat{\mathbf{y}}$, $\mathbf{H}^i \| -\hat{\mathbf{z}}$. The internal electric field to first order is:

$$\mathbf{E} = \mathbf{E}_0 + jk\mathbf{E}_1 = (A_2 + jk D_3 x)\hat{\mathbf{y}} + jk G_3 y\hat{\mathbf{x}} \quad (59)$$

where

$$A_2 = [(abc/2)(\epsilon - 1)I_b + 1]^{-1} \quad (60)$$

$$D_3 = -1/2 - D_c \quad (61)$$

$$G_3 = 1/2 - D_c \quad (62)$$

$$D_c = \frac{2 - (a^2 - b^2)(\epsilon - 1)\, abc\, I_{ab}}{4 + 2(\epsilon - 1)(a^2 + b^2)\, abc\, I_{ab}} \quad (63)$$

and I_{ab} is given by equation (47).

3.2.3. *KHE polarization.* The incident fields for this polarization are $\mathbf{E}^i \| \hat{\mathbf{z}}$, $\mathbf{H}^i \| \hat{\mathbf{y}}$. The internal electric field to first order is:

$$\mathbf{E} = \mathbf{E}_0 + jk\mathbf{E}_1 = (A_3 + jk D_2 x)\hat{\mathbf{z}} + jk G_2 z\hat{\mathbf{x}} \quad (64)$$

where

$$D_2 = -(1/2 + D_b) \quad (65)$$

$$G_2 = (1/2 - D_b) \quad (66)$$

$$D_b = \frac{2 - (\epsilon - 1)(a^2 - c^2)\, abc\, I_{ca}}{4 + 2(\epsilon - 1)(a^2 + c^2)\, abc\, I_{ca}} \quad (67)$$

$$A_3 = [(abc/2)(\epsilon - 1)I_c + 1]^{-1} \quad (68)$$

3.2.4. *HEK polarization.* The incident fields are $\mathbf{E}^i \| \hat{\mathbf{y}}$, $\mathbf{H}^i \| \hat{\mathbf{x}}$. The internal electric field to first order is:

$$\mathbf{E} = \mathbf{E}_0 + jk\mathbf{E}_1 = (A_2 + jk B_1 z)\hat{\mathbf{y}} + jk C_1 y\hat{\mathbf{z}} \quad (69)$$

where

$$B_1 = -1/2 - B_a \quad (70)$$

$$C_1 = 1/2 - B_a \quad (71)$$

$$B_a = \frac{2 + (\epsilon - 1)(b^2 - c^2)\, abc\, I_{bc}}{4 + 2(\epsilon - 1)(b^2 + c^2)\, abc\, I_{bc}} \quad (72)$$

3.2.5. *HKE polarization.* The incident fields for this polarization are $\mathbf{E}^i \| \hat{\mathbf{z}}$, $\mathbf{H}^i \| \hat{\mathbf{x}}$. The internal electric field to first order is:

$$\mathbf{E} = \mathbf{E}_0 - jk\mathbf{E}_1 = (A_3 - jk D_1 y)\hat{\mathbf{z}} - jk G_1 z\hat{\mathbf{y}} \quad (73)$$

where

$$D_1 = 1/2 - D_a \quad (74)$$

$$G_1 = -1/2 - D_a \quad (75)$$

$$D_a = -\frac{2 - (\epsilon - 1)(b^2 - c^2)\, abc\, I_{bc}}{4 + 2(\epsilon - 1)(b^2 + c^2)\, abc\, I_{bc}} \quad (76)$$

A_3 and I_{bc} are given previously.

The expressions for the first-order electric fields inside the ellipsoid for each of the six polarizations will be used in the next section to calculate the SAR of the ellipsoid.

4. CALCULATION OF AVERAGED SPECIFIC ABSORPTION RATE

Expressions for the first-order averaged SAR and the spatial variation of the SAR inside the ellipsoid are found by using the first-order internal electric fields as given in the previous section. The distribution of the SAR inside the ellipsoid is given by:

$$P(x, y, z) = (1/2)\, \sigma\, \mathbf{E} \cdot \mathbf{E}^* \quad (77)$$

where * denotes the complex conjugate. The averaged SAR is given by the volume integral

$$P_{av} = (1/V) \int_{z=-c}^{c} \int_{x=-a}^{a} \int_{-f(x,z)}^{f(x,z)} P(x,y,z)\, dx\, dy\, dz \quad (78)$$

where

$$f(x,z) = b[1 - (x^2/a^2) - (z^2/c^2)]^{1/2}$$

and $V = 4\pi abc/3$ is the volume of the ellipsoid.

The final results for the six polarizations are:

EKH

$$P(x,y,z) = (1/2)\sigma E^2 [(A_1 - jkB_3 y)(A_1 - jkB_3 y)^* + k^2 C_3 C_3^* x^2] \quad (79)$$

$$P_{av} = (1/2)\sigma E^2 [A_1 A_1^* + (k^2/5)(b^2 B_3 B_3^* + a^2 C_3 C_3^*)] \quad (80)$$

EHK

$$P(x,y,z) = (1/2)\sigma E^2 [(A_1 - jkC_2 z)(A_1 - jkC_2 z)^* + k^2 B_2 B_2^* x^2] \quad (81)$$

$$P_{av} = (1/2)\sigma E^2 [A_1 A_1^* + (k^2/5)(a^2 B_2 B_2^* + c^2 C_2 C_2^*)] \quad (82)$$

$$KEH \begin{cases} P(x,y,z) = (1/2)\sigma E^2 [(A_2 + jkD_3x)(A_2 + jkD_3x)^* + k^2 G_3 G_3^* y^2] & (83) \\ P_{av} = (1/2)\sigma E^2 [A_2 A_2^* + (k^2/5)(a^2 D_3 D_3^* + b^2 G_3 G_3^*)] & (84) \end{cases}$$

$$KHE \begin{cases} P(x,y,z) = (1/2)\sigma E^2 [(A_3 + jkD_2x)(A_3 + jkD_2x)^* + k^2 G_2 G_2^* z^2] & (85) \\ P_{av} = (1/2)\sigma E^2 [A_3 A_3^* + (k^2/5)(a^2 D_2 D_2^* + c^2 G_2 G_2^*)] & (86) \end{cases}$$

$$HEK \begin{cases} P(x,y,z) = (1/2)\sigma E^2 [(A_2 + jkB_1z)(A_2 + jkB_1z)^* + k^2 C_1 C_1^* y^2] & (87) \\ P_{av} = (1/2)\sigma E^2 [A_2 A_2^* + (k^2/5)(c^2 B_1 B_1^* + b^2 C_1 C_1^*)] & (88) \end{cases}$$

$$HKE \begin{cases} P(x,y,z) = (1/2)\sigma E^2 [(A_3 - jkD_1y)(A_3 - jkD_1y)^* + k^2 G_1 G_1^* z^2] & (89) \\ P_{av} = (1/2)\sigma E^2 [A_3 A_3^* + (k^2/5)(b^2 D_1 D_1^* + c^2 G_1 G_1^*)] & (90) \end{cases}$$

where E is the peak value of the incident electric field and has been assumed to be unity in the previous section. All of the other parameters appearing in equations (79) through (90) are given in Section 3.

As $b \to c$ (the ellipsoid becomes a spheroid) and as $a \to b \to c$ (the ellipsoid becomes a sphere), the above equations reduce to those for the spheroid and the sphere, respectively.

Expressions for the averaged SAR for the conductor ellipsoidal model have been reported previously [*Massoudi et al.,* 1977a]. The results of the calculation of averaged SAR in the conductor model and in the dielectric ellipsoidal model are compared in this section.

Figures 2 through 4 show the averaged SAR in the two models for a monkey-sized ellipsoid as a function of the conductivity, with the real part of the complex dielectric constant as a parameter, for *EKH* polarization at frequencies of 10, 30, and 100 MHz. The values of ϵ_1 given in the figures are those for muscle tissue and for saline solution at different frequencies. These values are: 210, 160, 110, 68, and 52 for muscle tissue at frequencies of, respectively, 5, 10, 30, 100, and 500 MHz [*Johnson et al.,* 1975], and 78 for saline solution at frequencies of 0.1-100 MHz [*Von Hippel,* 1954]. Note that the averaged SAR in the two models is the same if the imaginary part of the complex dielectric constant is about a factor of five larger than its real part, i.e., if

$$\epsilon_2/\epsilon_1 = \sigma/\omega\epsilon_0\epsilon_1 \geqslant 5 \qquad (91)$$

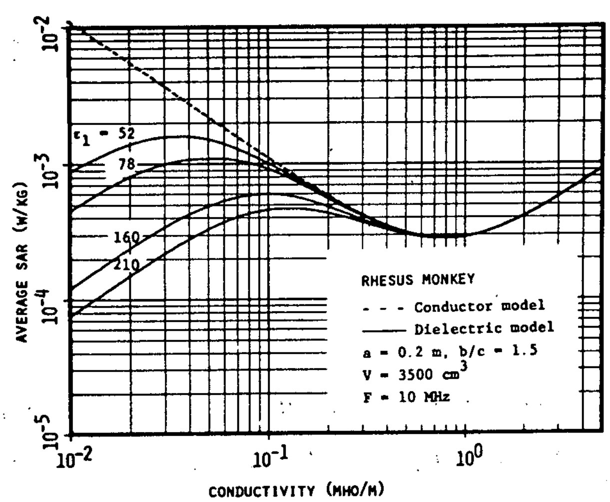

Fig. 2. Averaged specific absorption rate in the ellipsoidal model of a rhesus monkey as a function of the complex dielectric constant at 10 MHz. Incident power density is 1 mW/cm^2.

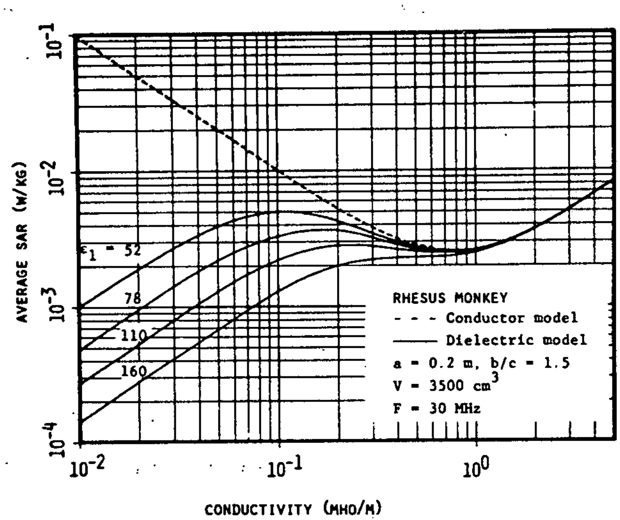

Fig. 3. Averaged specific absorption rate in the ellipsoidal model of a rhesus monkey as a function of the complex dielectric constant at 30 MHz. Incident power density is 1 mW/cm^2.

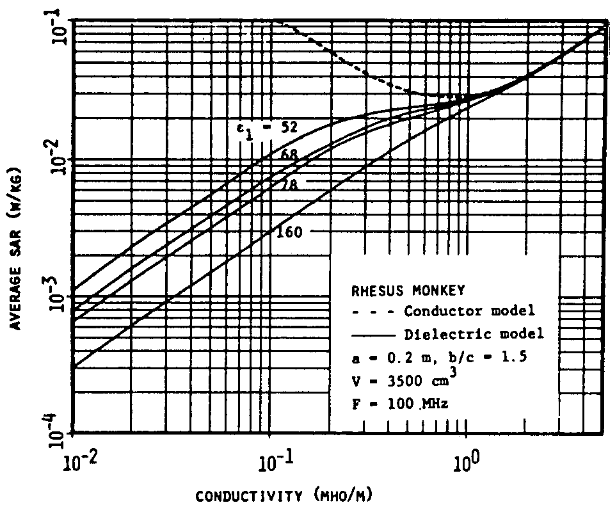

Fig. 4. Averaged specific absorption rate in the ellipsoidal model of a rhesus monkey as a function of the complex dielectric constant at 100 MHz. Incident power density is 1 mW/cm^2.

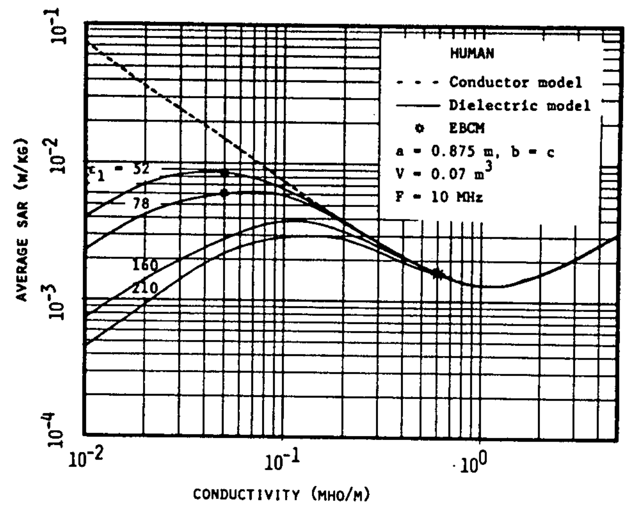

Fig. 5. Averaged specific absorption rate in the prolate spheroid model of an average man as a function of the complex dielectric constant at 10 MHz. Incident power density is 1 mW/cm^2.

It is also interesting to note, referring again to Figures 2-4, that the separation between the two models for a given ϵ_1 is a function of frequency. This would be expected because of the dependence of ϵ_2 on frequency, as stated in condition (91). Calculations have been carried out for the five other polarizations at several frequencies below 100 MHz and the results are qualitatively similar. The SAR in the conductor model at zero conductivity is infinite, whereas the dielectric model gives an SAR equal to zero for zero conductivity.

The first-order averaged SAR calculations in the prolate spheroidal ($b = c$) model of man have been compared with those of the EBCM; the results for the electric polarization (incident E field parallel to the major axes of the spheroid) are shown in Figure 5. It can be seen from Figure 5 that the results of the conductor model agree with those of the EBCM only for higher values of conductivity ($\sigma/\omega\epsilon_0 > \epsilon_1$) while the results of the dielectric model are in good agreement with those of the EBCM for all values of conductivity. This emphasizes that the theoretical results obtained in the conductor model, as described by *Durney et al.* [1975], are valid only if $\sigma/\omega\epsilon_0 \gg \epsilon_1$.

Figure 6 shows the averaged SAR in an ellipsoidal model of a rhesus monkey for the six polarizations as a function of frequency. Note that there is a strong orientational effect with the *EKH* polarization, resulting in approximately one order of magnitude increase in the averaged SAR as compared with the *HKE* polarization. The dependence of the averaged SAR on orientation of the body with respect to the incident electromagnetic fields has been described previously [*Massoudi et al.*, 1977b] and has been confirmed qualitatively by *Gandhi* [1975].

5. SUMMARY AND CONCLUSIONS

Perturbation theory has been applied to obtain the first-order solution of the internal electric fields and the electromagnetic SAR in a dielectric ellipsoidal model. The SAR in the conductor and in the dielectric ellipsoidal models of a rhesus monkey are compared. The two models give the same results if the conduction current in the body is much larger than the displacement current. The results obtained by perturbation theory for both the dielectric and the conductor prolate spheroidal ($b = c$) models of man have been compared with those of the EBCM. The results of the conductor model agree with those of the EBCM only for

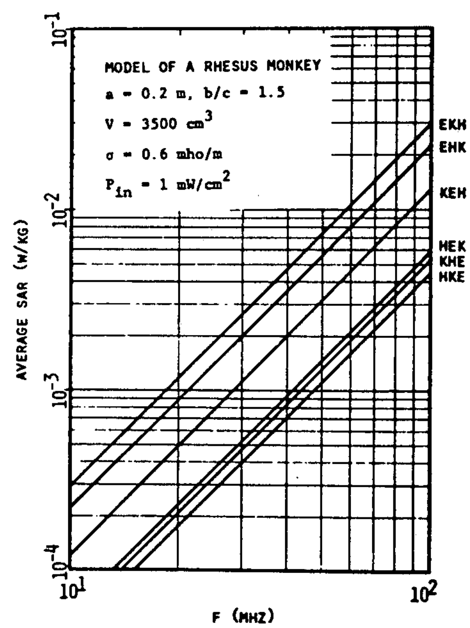

Fig. 6. Averaged specific absorption rate in the ellipsoidal model of a sitting monkey for the six standard polarizations. $a = 0.2$ m, $V = 0.0035$ m^3, $b/c = 2$, $\sigma = 0.6$ mho/m, $P_{in} = 1$ mW/cm^2.

higher values of conductivity, while the results of the dielectric model are in good agreement with those of the EBCM for all values of conductivity.

Since it has been found necessary to use values of tissue conductivity that are below the range of validity of the conductor model, the dielectric model is essential in such cases for accurate theoretical modeling.

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