ELECTRIC FIELD PROBES AND THEIR APPLICATION IN EMC

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The use of the linear antenna as a probe to measure both the constitutive parameters of material media and the electric field interior to such media is reviewed. The advantages of using probes with a particular geometry (i.e., electrically short, resonant, bare or insulated) for specific applications are discussed. Applications in the areas of geophysics and biophysics are cited.

Introduction

The properties and effects of electromagnetic radiation in material media are of fundamental importance in the area of electromagnetic compatibility. Among the electromagnetic properties of interest are the constitutive parameters of the medium since they completely characterize the interaction of the electromagnetic field with the medium at the macroscopic level. Also of interest is the distribution of the electric field in the medium since the effects of the electromagnetic radiation in the medium, like heating, are a function of the electric field strength. In many problems of practical interest these quantities, the constitutive parameters and distribution of electric field, cannot be calculated simply from the equations of field theory and must be measured experimentally using suitably designed probes. In this paper the use of the linear antenna as a probe is examined with these measurements in mind.

Specific applications arise in the areas of geophysics and biophysics where probes can be used to measure the electrical properties of material media such as rock, soil, ionospheric plasma and biological tissue. Receiving probes are also of use in these areas. For example, a receiving probe embedded in the earth can be used to measure the penetration of electromagnetic pulses after a nuclear explosion. Probes inserted in an irradiated biological specimen can be used to measure the electric field so that any structural damage to the tissue can be correlated with the local electric field strength. Other applications for receiving probes include measurement of the near fields of antenna systems and leaky microwave ovens.

In Situ Measurement of the Constitutive Parameters of Material Media

The macroscopic electrical properties of a material medium are completely described by three scalar constitutive parameters, viz.. the real effective conductivity σ_e, the real effective permittivity (dielectric constant) $\epsilon_e = \epsilon_r\epsilon_0$, and the real effective permeability $\mu_e = \mu_r\mu_0$, which is assumed to be that of free space for the materials considered here ($\mu_r = 1.0$). These quantities are related to the complex conductivity $\sigma = \sigma' + i\sigma''$ and the complex permittivity $\epsilon = \epsilon' + i\epsilon''$ by the relations

$$\sigma_e = \sigma' + \omega\epsilon'' \quad , \quad \epsilon_e = \epsilon_r\epsilon_0 = \epsilon' - \sigma''/\omega \tag{1}$$

where a harmonic time dependence of the form $e^{-i\omega t}$ is assumed. It is often convenient to introduce the loss tangent

$$p = \sigma_e/\omega\epsilon_e \tag{2}$$

and the wave number for a plane wave propagating in the medium

$$k = \beta + i\alpha = \omega[\mu_0\epsilon_e(1 + ip)]^{1/2}$$
$$= \omega(\mu_0\epsilon_e)^{1/2}f(p)[1 + ip/2f^2(p)] \tag{3}$$

$$\alpha/\beta = p/2f^2(p) \tag{4}$$

where
$$f(p) = \{(1/2)[1 + (1 + p^2)^{1/2}]\}^{1/2} \tag{5}$$

The input admittance $Y = G - iB$ or impedance $Z = R - iX$ of a bare, thin wire ($a/h << 1$), linear antenna embedded in a medium is a function of the constitutive parameters of the medium. See Figure 1 for the geometry of the bare dipole antenna. Under certain conditions the measured input admittance or impedance of the antenna can be used to determine the constitutive parameters of the surrounding medium.

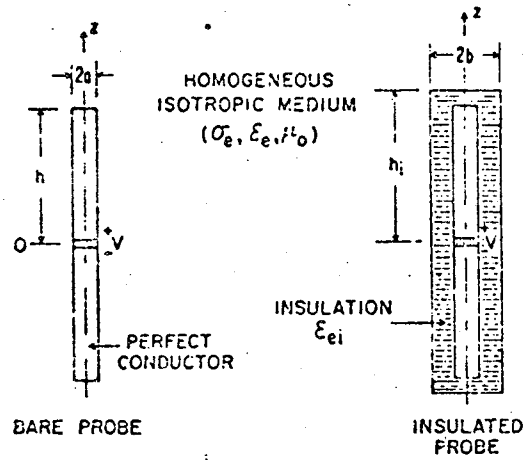

Fig. 1: Details of bare and insulated probes.

Electrically Short Probe

The bare linear antenna is electrically short when

$$|kh| < 1 \tag{6}$$

The input admittance of the electrically short antenna has been theoretically determined by using an expansion for the current distribution on the antenna in terms of powers of kh (terms to order k^3h^3 are retained in the expression) [1]. For the purpose of obtaining the ϵ_r and p of the surrounding medium from the input admittance, it is convenient to use the following normalized quantities

$$B/B_0 = (\beta/\beta_0)^2\{(1 - \alpha^2/\beta^2) + (1/3)\beta^2h^2F(1 - 6\alpha^2/\beta^2$$
$$+ \alpha^4/\beta^4) - [\beta^3h^3/3(\Omega - 3)](\alpha/\beta)(5 - 10\alpha^2/\beta^2$$
$$+ \alpha^4/\beta^4) + \ldots\}\{1 + (1/3)\beta_0^2h^2F + \ldots\}^{-1} \tag{7}$$

$$G/B = (1-\alpha^2/\beta^2)^{-1}\{2(\alpha/\beta)(1-\alpha^2/\beta^2) + (8/3)\beta^2h^2 \times F(\alpha^3/\beta^3) + [\beta^3h^3/3(\Omega-3)](1-\alpha^2/\beta^2 - 5\alpha^4/\beta^4 - 3\alpha^6/\beta^6) + \ldots\} \quad (8)$$

where

$$\Omega = 2\,\ell n(2h/a)\ ,\ F = 1 + [1.08/(\Omega - 3)] \quad (9)$$

B_0 is the susceptance of the antenna in free space and $\beta_0 = 2\pi/\lambda_0$ is the propagation constant in free space. By substituting measured values of B/B_0 and G/B into the corresponding theoretical expressions in equations (7) and (8), these can be solved numerically or graphically to determine ϵ_r and p of the surrounding medium. This is illustrated in Figure 2 where B/B_0 and G/B are plotted for an electrically short antenna ($h/\lambda_0 = 3 \times 10^{-3}$) with ϵ_r and p as parameters. Note that $p \doteq G/B$ and $\epsilon_r \doteq B/B_0$ for low values of ϵ_r and p. This is what one would expect from a quasi-stationary analysis which is equivalent to retaining only the terms independent of βh in equations (7) and (8). For the higher values of ϵ_r and p, G/B and B/B_0 depart from this simple behavior. This is the result of retaining the higher powers of βh in the theoretical expressions.

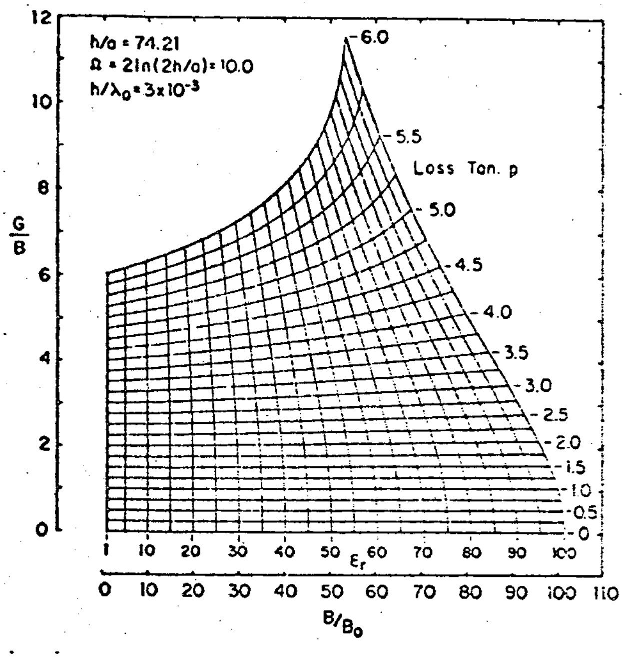

Fig. 2: B/B_0 and G/B of an electrically short probe for fixed values of the relative dielectric constant ϵ_r and loss tangent p of the surrounding medium.

The electrically short antenna has been used successfully as a probe to measure the constitutive parameters of ionospheric and laboratory plasmas [2],[3] and geological material [4]. Figure 3 illustrates the accuracy of the technique. Values of the relative dielectric constant ϵ_r for liquid mixtures with low loss ($p \doteq 0$) measured with an electrically short probe are compared with values of ϵ_r measured by an independent method. The agreement between the measured values is seen to be good over the entire range of dielectric constant used ($2.4 \leq \epsilon_r \leq 80$).

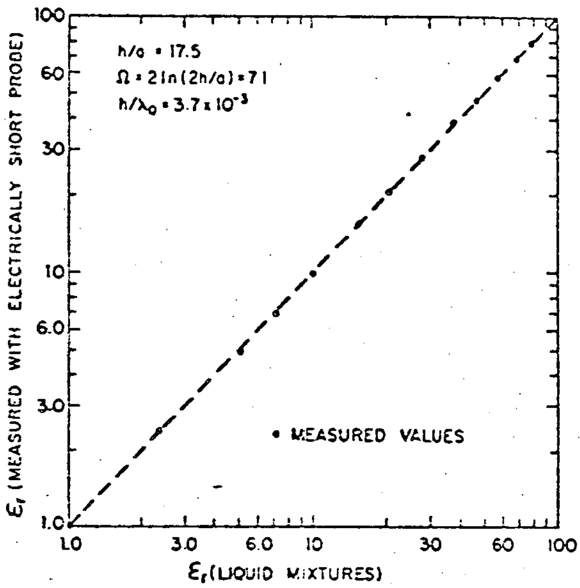

Fig. 3: Relative dielectric constant ϵ_r of liquid mixtures with low loss measured with an electrically short probe.

Resonant Probe

As the frequency at which the electrical properties of a material are to be measured is increased, the physical length h of an electrically short probe must be decreased in order to satisfy the inequality $|kh| < 1$. For any given material, there is a maximum frequency at which the electrically short probe can be used; to go beyond this frequency would require a decrease in the probe length to an impractical value. At these higher frequencies it is convenient to use electrically longer antennas as probes. One method which has been employed uses an antenna of resonant length h_r, which is defined as the shortest length for which the input reactance X of the antenna is zero [5]. The procedure for this method is to insert the probe in the material to be measured and then to adjust the antenna length or operating frequency to produce resonance. The resonant length h_r/λ_0 and the input resistance at resonance R_r are measured. The electrical properties of the medium surrounding the antenna are then determined from a graphical representation of the theoretical values for h_r/λ_0 and R_r with ϵ_r and p as parameters; see Figure 4.

Any measuring technique is usually applicable over a limited frequency range and a combination of techniques must be used when a broad frequency range is to be covered. The electrically short and resonant probe techniques are suitable for covering adjacent frequency ranges. In addition, the measuring procedure can be simplified since the same probe elements can be used for both techniques and only the frequency varied to change the electrical length of the probes.

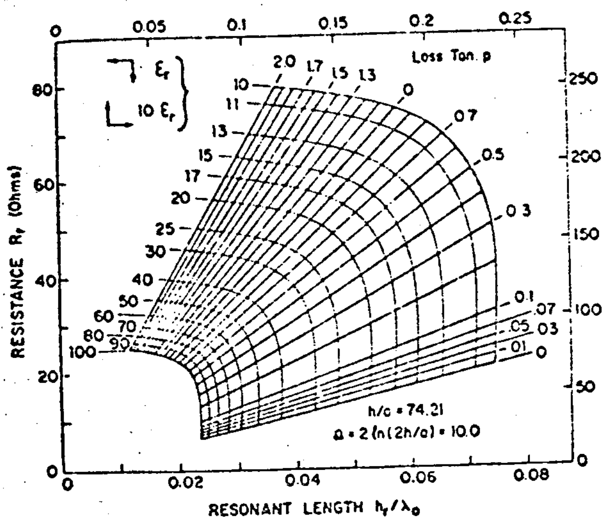

Fig. 4: Graphical representation of the resonant length and resistance at resonance for fixed values of relative dielectric constant ϵ_r and loss tangent p of the surrounding medium.

Figure 5 illustrates the use of both electrically short and resonant probes to measure the electrical properties of moist sand in the frequency range (20 MHz $\leq$ f $\leq$ 400 MHz).

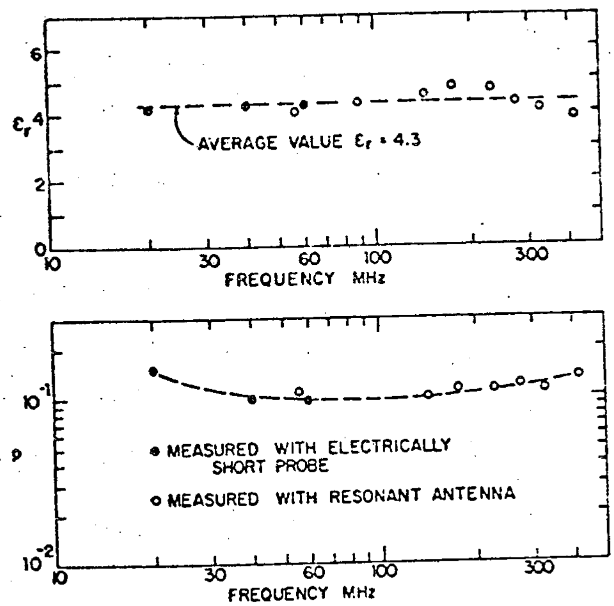

Fig. 5: Relative dielectric constant ϵ_r and loss tangent p of moist sand as a function of frequency.

Insulated Probes

The insulated probe is formed by adding a concentric dielectric sheath of radius b and length h_i to the bare probe; see Figure 1. In the following analysis the sheath is assumed to be lossless and has the electrical constitutive parameters ϵ_{ei}, σ_{ei} = 0, and μ_0.

Electrically short probe. The addition of the insulating sheath can greatly alter the effect that the electrical properties of the external medium ϵ_r, σ_e have on the probe input admittance. This is illustrated in Figure 6 where the normalized probe input capacitance C/C_0 of electrically short, bare and insulated (ϵ_{ri} = 1.0, b/a = 2, 4) probes is plotted as a function of the dielectric constant of the external medium, ϵ_r. C_0 is the capacitance of the probe in air. The calculations for Figure 6 were made using a simple quasi-stationary theory for the capacitance of the electrically short insulated probe [6] and the external medium was assumed to be lossless, p = 0. Note that the input capacitance of the insulated probe is insensitive to the dielectric constant of the external medium, ϵ_r, when the ratio ϵ_r/ϵ_{ri} is large, whereas the capacitance of the bare probe is a linear function of ϵ_r.

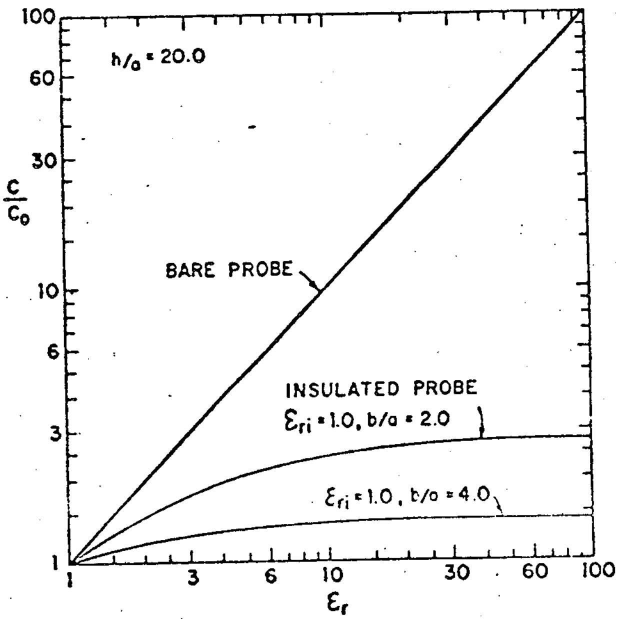

Fig. 6: Normalized input capacitance of electrically short, bare and insulated probes as a function of the relative dielectric constant ϵ_r of the external medium.

This analysis suggests that the electrically short insulated probe is only useful for measuring the electrical properties of the surrounding medium when they are not much different from those of the insulation. For example, electrically short insulated probes have been used successfully in boreholes to measure the in situ dielectric properties of Antarctic ice [7]. The electrical properties of the insulation and of the ice at the frequencies used were: $\epsilon_{ri} \doteq 2.1$, $4 \leq \epsilon_r \leq 16$, $0 \leq p \leq 2$.

Resonant probe. The insulated linear antenna of general length has not been analyzed for arbitrary electrical properties of the insulation and external medium. The theoretical analysis that is available applies when the ratio $|k|^2/\beta_i^2$ is not too small; k is the wave number in the external medium and β_i is the propagation constant in the lossless dielectric insulation [8]. The current distribution on the antenna is sinusoidal and similar to that on an open-circuited section of transmission line,

$$I(z) = -\frac{i\pi\omega\varepsilon_{ei}V}{k_L \ln(b/a)} \frac{\sin k_L(h - |z|)}{\cos k_L h} \qquad (10)$$

where k_L is the wave number on an infinitely long antenna with the same cross section. The wave number

$$k_L = \beta_L + i\alpha_L \qquad (11)$$

is the complex root ζ of the following equation:

$$(\beta_i^2/\xi_1)H_0^{(1)}(b\xi_2)[H_0^{(1)}(a\xi_1)J_1(b\xi_1) - J_0(a\xi_1) \times H_1^{(1)}(b\xi_1)] - (k^2/\xi_2)H_1^{(1)}(b\xi_2)[H_0^{(1)}(a\xi_1) \times J_0(b\xi_1) - J_0(a\xi_1)H_0^{(1)}(b\xi_1)] = 0 \qquad (12)$$

where

$$\xi_1 = (\beta_i^2 - \zeta^2)^{1/2} \; ; \; \xi_2 = (k^2 - \zeta^2)^{1/2} \qquad (13)$$

Experimental investigations have shown that equation (10) is an accurate representation for the current distribution on the antenna when the ratio $|k|^2/\beta_i^2 \geq 14$ [9],[10].

The resonant length for the insulated antenna is obtained by determining the value h_r/λ_0 for which $\mathrm{Im}[I(0)] = 0$. This is equivalent to solving for the root x of the following equation:

$$\sin[4\pi(\beta_L/\beta_0)x] = (\alpha_L/\beta_L)\sinh[4\pi(\beta_L/\beta_0) \times (\alpha_L/\beta_L)x] \qquad (14)$$

In Figure 7 the resonant length of bare and insulated ($\varepsilon_{ri} = 1.0$, $b/a = 2, 4$) probes is plotted as a function of the relative dielectric constant ε_r of the external medium, which is assumed to be lossless, $p = 0$.

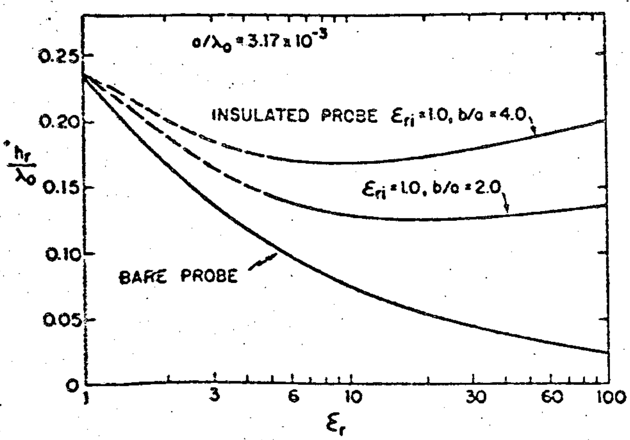

Fig. 7: Resonant length h_r/λ_0 of bare and insulated probes as a function of the relative dielectric constant ε_r of the external medium.

Several interesting points are illustrated in Figure 7. When the dielectric constants of the insulation and external medium are the same, the bare and insulated probes must have the same resonant length, $h_r/\lambda_0 \doteq 0.235$. The resonant length of the bare probe is approximately proportional to $1/\varepsilon_r^{1/2}$. The resonant length of the insulated probe is less dependent on ε_r than for the bare probe: h_r/λ_0 is decreasing at lower values of ε_r, becomes fairly constant at intermediate values ($\varepsilon_r \sim 10$), and then begins to increase at higher values of ε_r. Eventually, at very high values of ε_r, h_r/λ_0 must approach the resonant length of a section of transmission line terminated in an open circuit, $h_r/\lambda_0 \doteq 0.25$.

The resonant length of the insulated probe is fairly independent of the dielectric constant of the external medium over a broad range of values of ε_r and the same resonant length can be obtained for two values of ε_r. These facts suggest that the resonant insulated probe would not be useful for determining the electrical properties of the surrounding medium. A possible exception is the case in which the electrical properties of the insulation are chosen to be similar to those of the medium to be measured.

Measurement of the Electric Field in Material Media

A knowledge of the electric field interior to a material body is often required - for example, to determine if interference or a possible hazard exist. Most material bodies or practical interest are spatially inhomogeneous in the sense that the electromagnetic constitutive parameters (ε_e, σ_e) are not uniform throughout. This is certainly true for bodies like the earth and biological specimens. Ideally, a probe is needed that will measure the electric field at different points in such a body in a manner independent of the local constitutive parameters. The response from the probe, i.e., the terminal voltage V, should be proportional to the local electric field $\vec{E} = E\hat{e}$:

$$V = K_e E \qquad (15)$$

and the proportionality constant K_e should be independent of the constitutive parameters of the surrounding medium [6]. A probe with this kind of response eliminates the need for specific knowledge of the constitutive parameters of the material at each location of the probe. A separate calibration is not required for each medium in which the probe will be used; a single calibration in one medium is sufficient.

Electrically Short Electric Field Probes

Electrically short linear antennas are attractive for this application since their small physical size allows high spatial resolution with a minimum field perturbation. The voltage V developed across the terminals of an electrically short probe is a measure of the component of the local electric field parallel to the axis of the probe, $\vec{E}_z = E_z\hat{z}$. The relationship between V and E_z can be expressed with the aid of the Thévenin equivalent circuit for the probe (see Figure 8)

$$V = K_e E_z = -2h_e[Y/(Y + Y_\ell)]E_z \qquad (16)$$

where $2h_e$ is the effective height of the probe, Y is the input admittance of the probe to an applied voltage, and $Y_\ell = G_\ell - iB_\ell$ is the load admittance connected to the terminals of the probe.

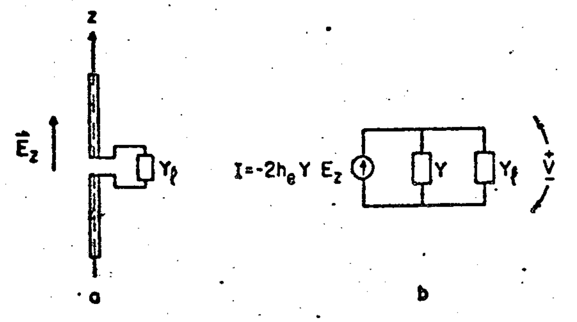

Fig. 8: Loaded receiving probe and its equivalent circuit.

For electrically short, bare and insulated probes the effective height is approximately h,

$$K_e \doteq -h[Y/(Y + Y_\ell)] \qquad (17)$$

The desired response for the probe, i.e., having K_e independent of the constitutive parameters (ε_r, σ_e) of the surrounding medium, can be obtained by either 1) making the load admittance $|Y_\ell| << |Y|$ so that $K_e \doteq -h$, or 2) making Y independent of ε_r and σ_e.

The admittance of the electrically short bare probe is a function of ε_r and σ_e; therefore, its response can be made independent of ε_r and σ_e only if a load admittance such that $|Y_\ell| << |Y|$ is used. Note, however, that the input admittance of the short probe can be a very small capacitance, making it difficult to satisfy the inequality. As previously illustrated, the input admittance of the electrically short insulated probe can be made fairly independent of ε_r and σ_e for a broad range of values for these parameters (see Figure 6). The response of the insulated receiving probe will be independent of the constitutive parameters of the external medium over the same range of values for ε_r and σ_e. This is true no matter what value of load admittance Y_ℓ is used.

The characteristics of the probes are illustrated in Figure 9 where the magnitude of the normalized received voltage

$$V_n(\varepsilon_r) = K_e(\varepsilon_r)/K_e(\varepsilon_r = 80) \qquad (18)$$

is plotted as a function of the relative dielectric constant of the external medium. Measured and theoretical values are shown for a bare probe and a similar insulated probe with a teflon sheath ($\varepsilon_{ri} \doteq 2.1$, $b/a = 3.28$). The external media used for the measurements were liquid mixtures with low loss, $p \doteq 0$.

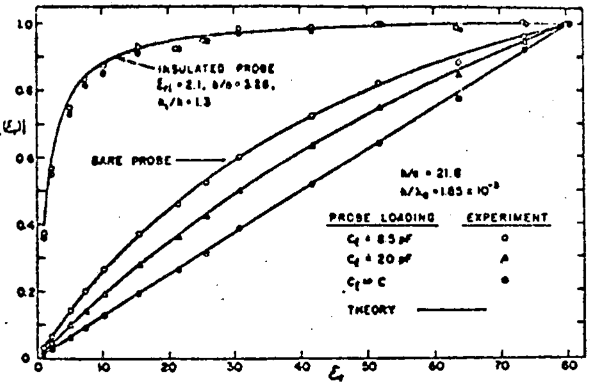

Fig. 9: Normalized electric field response of electrically short, bare and insulated probes as a function of the relative dielectric constant ε_r of the external medium and the probe loading C_ℓ.

The normalized response of the bare probe is seen to be a function of the relative dielectric constant of the external medium, ε_r, and the admittance of the load, $Y_\ell = -i\omega C_\ell$. The response of the insulated probe is fairly uniform for the higher values of ε_r, showing only a 10% variation for $\varepsilon_r \geq 12$.

The received voltage of a single electrically short probe is proportional to the component of the local electric field parallel to the probe axis. Combinations of electrically short probes can be arranged to provide a uniform response to electric fields in arbitrary directions. Referring to Figure 10(a), three orthogonal probes parallel to the x, y and z axes will have the following individual received voltages when placed in a local electric field with arbitrary orientation $\vec{E} = E\hat{e}$,

$$V_x = K_e E \sin\theta \cos\phi$$

$$V_y = K_e E \sin\theta \sin\phi \qquad (19)$$

$$V_z = K_e E \cos\theta$$

If signals proportional to the square of the amplitudes of these voltages are detected and the three components summed (see Figure 10(b)), the result is a signal proportional to $|\vec{E}|^2$,

$$|V_x|^2 + |V_y|^2 + |V_z|^2 = (K_e)^2|\vec{E}|^2 \qquad (20)$$

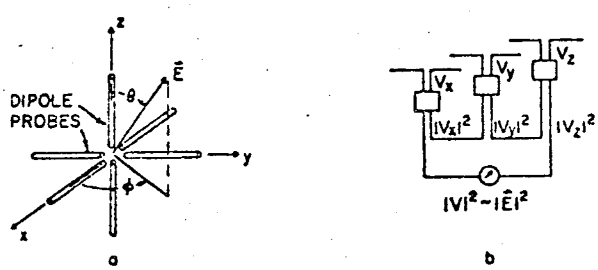

Fig. 10: (a) Orthogonal dipoles in an electric field with arbitrary orientation; (b) Schematic representation of signal detection circuitry.

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