# 7 – 7 ELECTROTONIC SOLUTION OF RECTANGULAR ELECTRICAL ANESTHESIA CURRENTS APPLIED TO MODEL NEURONS

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INTRODUCTION

Previous studies[1] have demonstrated that inion-asion electroanesthesia currents are capable of ren-ering Macaque monkeys unresponsive to peripheral timuli. The currents were typically 100 per second, .5 ms duration, rectangular pulses biased above zero.

Animal responsiveness was reduced at peak rec-ıngular (pr) levels of 12 ma in conjunction with a dc urrent of 7 ma. A response to sciatic stimuli could ɔt be obtained at pr levels from 20 to 25 ma in con-ınction with 7 ma dc. Simultaneous brain and periph-ral evoked cortical[2] potentials were markedly reduced r obliterated.

The empirical results are interpreted in light of a ɔrtical soma-dendrite model, subjected to a repeti-ve rectangular current similar to those used in our :udies. An electrotonic solution to a unit rectangular ırrent provides the general solution. The results are ɘlated to the efficacy of various rectangular currents. mpirical cortical current density[3] coupled with the .odel system suggest neuronal polarization.

SOMA-DENDRITE MODEL

The model assumes that a current is impressed ɔon the soma of a lumped soma-dendrite[4] neuron. he derivation of the electrotonic equation for a cylin-rical system is well established in the theory of axons. may be expressed as,

1) $$\frac{\delta^2 V}{\delta X^2} = \left(\frac{\delta V}{\delta T} + V\right)$$ where the normal-

ed variables $X = x/\lambda$ and $T = t/\Upsilon$ . X = the longi-dinal distance along the cable, $V = V_m - E$ and $\Upsilon = r_m C_m$, $\lambda^2 = r_m/r_i$, and A = cable radius. The me constant $\Upsilon$ and space constant $\lambda$ are expressed in rms of their effective resistance. That is, $r_m$ is the ıunt resistance of the cable in 1 cm of length, $_m/\pi A^2$ and $r_i$ is the shunt resistance of the cyto-asm in 1 cm of fiber $R_i/2\pi A$. The resting potential and membrane potential $V_m$ are commonly expressed volts, $R_m$ in ohm $cm^2$ and $R_i$ in ohm cm.

For zero initial conditions and lim $X \longrightarrow \infty$, (X, T) ⟶ 0 the laplace transform of equation (1) be-ɔmes

2) $$V(X, S) = A_1 e^{-\sqrt{1+S}\,X}$$

here $A_1$ is a constant, and S is the transformed vari-ɔle T.

The model assumes that the soma is a lumped embrane of shunt resistance $r_s$ and shunt capacitance s. A current applied at the soma (X = O) may be con-idered as that which flows into the dendrite* plus the contribution to the soma, hence

3) $$i(O, t) = \frac{1}{r_s}\,(V + \delta V/\delta T) - \frac{1}{r_i \lambda}\,\delta V/\delta X$$

From equations (2) and (3) the transmembrane poten-tial for a unit rectangular current[5] of repetition rate f and pulse duration d becomes

4) $$V(X, T) = K \sum_{n=0}^{m} \Bigg[ e^{-X}\,\mathrm{erfc}\left(\frac{X}{2\sqrt{T_n}} - \sqrt{T_n}\right) - \frac{(B+1)}{(B-1)}\,e^{X}\,\mathrm{erfc}\left(\frac{X}{2\sqrt{T_n}} + \sqrt{T_n}\right) + \frac{2}{(B-1)}\,e^{BX+(B^2-1)T_n}\,\mathrm{erfc}\left(\frac{X}{2\sqrt{T_n}} + B\sqrt{T_n}\right)\Bigg] \cdot U(T_n)$$
$$- K \sum_{n=0}^{m} \Bigg[ e^{-X}\,\mathrm{erfc}\left(\frac{X}{2\sqrt{T'_n}} - \sqrt{T'_n}\right) - \frac{(B+1)}{(B-1)}\,e^{X}\,\mathrm{erf}\left(\frac{X}{2\sqrt{T'_n}} + \sqrt{T'_n}\right) + \frac{2}{(B-1)}\,e^{BX+(B^2-1)T'_n}\,\mathrm{erfc}\left(\frac{X}{2\sqrt{T'_n}} + B\sqrt{T'_n}\right)\Bigg]\, U(T'_n)$$

where K is a constant, U is the unit function, $T_n = \frac{t - nf}{\Upsilon}$,

$$T'_n = \left[\frac{t - (nf + d)}{\Upsilon}\right],$$

$$B = \frac{(A_d)}{2\,A_s^2}\sqrt{\frac{(A_d)}{2}\,\frac{R_m}{R_i}}$$ and erfc (y) =

$$= \frac{2}{\sqrt{\pi}} \int_y^{\infty} e^{-z^2}\,dz.$$

Thus, equation (4) gives the temporal transmembrane potential as a function distance from the soma for a repetitive unit rectangular current. The solution depends upon the parameter B which is a function $R_m/R_i$, dendrite ratius ($A_d$) and soma radius ($A_s$), where the lumped soma resistance is assumed to be $R_m/4\pi A_s^2$.

*A single, infinitely long, dendrite is assumed for the ıodel; however, it can be shown[4] that additional den-rites from neural samples the effective contribution is t most approximately 15% to 20%.

## 3. DISCUSSION

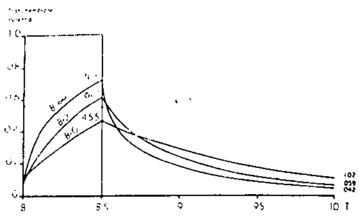

Figure 1 shows a plot of equation (4) at X =0). After eight times constants, little change is observed.

For purposes of discussion, d is chosen as $\tau/2$, consistent with f = 10 ms, and a pulse width of 2. 5 ms. A neuron time constant of 5 ms is assumed. The upper curve represents the B = $\infty$ solution, while the lower curve represents the B = 0 solution. A value B = 2 was chosen for the system. This can be shown to represent a class of dendrite to soma radii and $R_m/R_i$ ratios. For example, it is consistent with $R_m/R_i$ = 10 and $A_s$ = 10 $A_d$ for $A_d$ = 0. 35$\mu$ or, $A_s$ = 5$A_d$ for $A_d$ = 5. 0$\mu$.

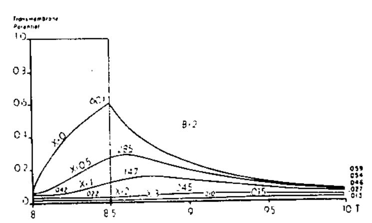

Figure 2 shows B = 2 response at various distances down the line. The numbers above each curve correspond to the approximate maximum levels.

A superimposed dc current similar to that used in studies would move the system response upward. Neglecting accommodation and tissue effects an increase in minimum and maximum polarizations proportional to pr-dc ratio and would be expected. The exact role of the separate components is difficult to determine. However, empirical observations have shown that smoother induction and better anesthesia are obtained with a superimposed dc current.

The roles of d and f versus $\tau$ are of considerable significance, since a compromise is sought between best anesthesia and minimum input energy.

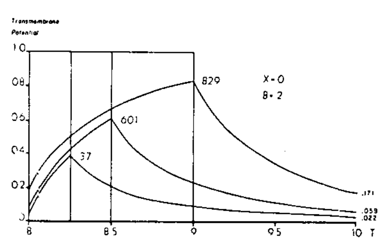


Figure 3 illustrates the effect of pulse width at X = O for B = 2. As the pulse width is increased from $\tau/4$ to $\tau$, an increase in the peak level considerably less than 4 to 1 is observed. Beyond $\tau/2$ the peak values change slowly with an increase in pulse width. In the limiting case (dc input equal to the peak current) an average input current equal to 4 times the $\tau/2$ curve would be needed to increase maximum level by 40%. Empirical observations have shown that approximately 3 to 4 times more direct current, compared to the average rectangular current, was necessary for animal unresponsiveness.

The model collaborates certain empirical results. It further suggests that the currents* may be capal of polarizing neuronal membranes sufficiently to a count for the physiologic findings. Synaptic influe and active processes will be included in future mo

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* Cortical pr values of 400$\mu$A/cm$^2$ have been observed[3] at the anesthetic level.