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Hot Spots Generated in Conducting Spheres by Electromagnetic Waves and Biological Implications

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Abstract—The distribution of the heating potential generated by an incident electromagnetic plane wave on a conducting sphere simulating the human head was investigated. It was found that for a sphere of 10-cm radius having the same electrical characteristics as those of biological tissues, no hot spots are generated inside. While at lower frequencies the heating is relatively uniform with some polarization effects, for frequencies above 1000 MHz only skin heating takes place. For a sphere of the same size but of conductivity of $\sigma = 10$ mmho/cm (which for $f > 1000$ is lower than that of biological tissues) hot spots occur inside for $f > 1000$ MHz. Intense hot spots also occur inside spheres of radius 5 cm having the same electrical characteristics as those of biological tissues in the frequency region of $250 \text{ MHz} < f < 2800 \text{ MHz}$.

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THE PROBLEM of heating biological tissues by electromagnetic waves, because of its importance in assessing potential electromagnetic health hazards, has received considerable attention in the literature. Schwan and co-workers [1]-[3] have studied the mechanism of heating biological tissues by electromagnetic fields. In particular, Anne *et al.* [4]-[6] have obtained estimates of electromagnetic heating of mankind by studying the total heat generated in conducting spheres and phantoms of the human body. The present communication presents a further contribution to this area by examining theoretically the distribution of the generated heating potential Ψ ($\Psi = \frac{1}{2}\sigma|E|^2$, σ is the conductivity, E is the electric field) inside a sphere that is immersed in an incident plane electromagnetic wave (see Fig. 1).

The electrical conductivity σ and relative dielectric

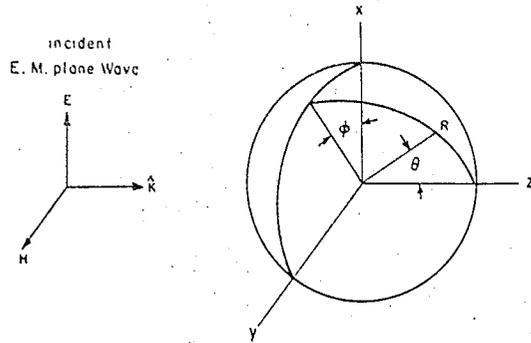


Fig. 1. Geometry of the problem.

constant ϵ_r were taken to be that of biological tissues with high water content such as muscle, brain matter, and body organs [7]. These properties can be expressed in close approximation in the following form [8], [9]

$$\sigma = 10 + \frac{550\epsilon_0\omega_0}{1 + (f/f_0)^2} \left(\frac{f}{f_0}\right)^2 \quad \text{in mmho/cm} \quad (1)$$

$$\epsilon_r = 5 + \frac{55}{1 + (f/f_0)^2} \quad (2)$$

where

$$\epsilon_0 = \frac{10^{-11}}{36\pi} \quad \text{in F/cm}, \quad f_0 = 20 \times 10^9 \text{ Hz.} \quad (3)$$

A useful measure of the degree of penetration of electromagnetic fields inside a conducting body is the penetration length L . The penetration length is defined as the distance at which the power density of an incident electromagnetic wave on a plane electromagnetic slab with propagation constant k_1 is reduced to $1/e$ of its value at the surface. One has

$$1/L = 2 \operatorname{Im} k_1 \quad (4)$$

where

$$k_1 = \frac{\omega}{c} \sqrt{\epsilon_r + i \frac{\sigma}{\epsilon_0\omega}} \quad (5)$$

and

$$\omega = 2\pi f, \quad c = 3 \times 10^{10} \text{ cm/s.} \quad (6)$$

Fig. 2 shows the penetration length and conductivity versus frequency. The heating potential can be obtained from the well-known expressions of the electric field inside a conducting sphere [10]. These are

$$\vec{E} = E_0 e^{-i\omega t} \sum_{n=1}^{\infty} i^n \frac{2n+1}{n(n+1)} (a_n^t \vec{m}_{01n}^{(1)} - i b_n^t \vec{n}_{e1n}^{(1)}) \quad (7)$$

where

$$\vec{m}_{01n}^{(1)} = \frac{1}{\sin \theta} j_n(k_1 R) P_n^1(\cos \theta) \cos \phi \hat{\theta} - j_n(k_1 R) \frac{\partial P_n^1}{\partial \theta} (\cos \theta) \sin \phi \hat{\phi} \quad (8)$$

$$\vec{n}_{e1n}^{(1)} = \frac{n(n+1)}{k_1 R} j_n(k_1 R) P_n^1(\cos \theta) \cos \phi \hat{r} + \frac{[k_1 R j_n(k_1 R)]'}{k_1 R} \frac{\partial P_n^1}{\partial \theta} (\cos \theta) \cos \phi \hat{\theta} - \frac{[k_1 R j_n(k_1 R)]'}{k_1 R \sin \theta} P_n^1(\cos \theta) \sin \phi \hat{\phi} \quad (9)$$

$$a_n^t = - \frac{i/\rho}{h_n^{(1)}(\rho) [N \rho j_n(N \rho)]' - j_n(N \rho) [\rho h_n^{(1)}(\rho)]'} \quad (10)$$

$$b_n^t = - \frac{iN/\rho}{[N \rho j_n(N \rho)]' h_n(\rho) - N^2 j_n(N \rho) [\rho h_n^{(1)}(\rho)]'} \quad (11)$$

$$\rho = k_2 A, \quad \text{where } A \text{ is the radius of the sphere, and } k_2 = 2\pi f/c$$

$$N = \frac{k_1}{k_2}, \quad \text{where } k_1 \text{ is the propagation constant of the material of the sphere.}$$

E_0 is the incident field of the sphere.

The above calculations were carried out with an IBM 360 computer. It was found by trial and error that the number of terms required for convergences of the above series is approximately $M = 2 |k_1 A|$.

The computer was programmed to search inside the sphere, find the maximum heating potential, and print it out. In this manner, it was possible to plot the heating potential of the hottest spot as a function of frequency.

The first case that was examined was that of a sphere of 10 cm. This was selected as being an idealization of the head of an adult. The results are shown in Fig. 3 where the surface heating potential of an infinite plane slab was also plotted for comparison. It was found that no hot spots occur inside the sphere. The maximum heating takes place always at the front surface. The results show that in the high-frequency region (i.e., $f > 1000$ MHz), the front heating potential approaches that of the plane infinite slab. This is in agreement with the geometrical approximation. Figs. 4-6 show the details of the heating potential distribution inside

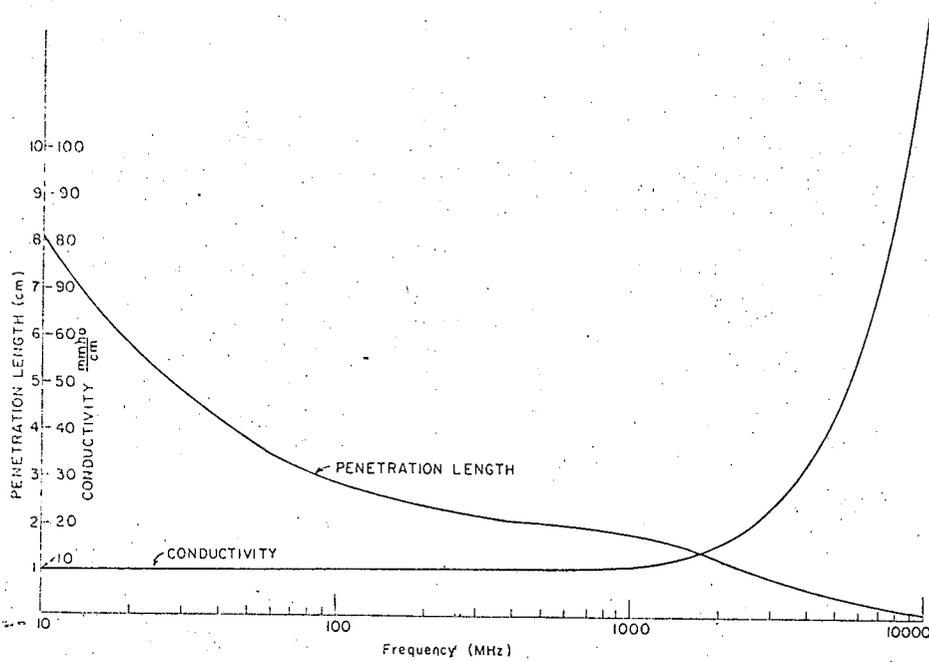


Fig. 2. Penetration length and conductivity versus frequency for an infinite plane slab having the same characteristics as those of biological tissues.

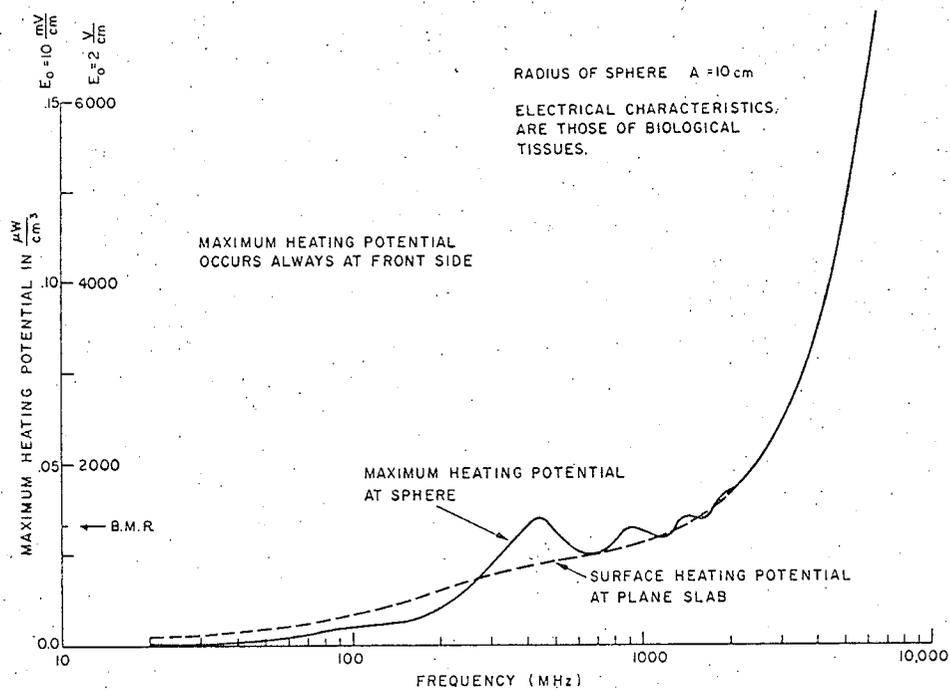


Fig. 3. Maximum heating potential for a sphere of 10-cm radius made of material with the same properties as those of biological tissues versus frequency.

and at the surface of the sphere. As expected from the larger penetration length values at the lower frequencies, the heating at the lower frequencies is more uniform, while preferential surface heating becomes increasingly pronounced at higher frequencies. It is interesting to note a marked polarization effect at lower frequencies. While the heating is uniform in the E plane, in the H plane at the surface in the transverse direction (i.e.,

$\phi = \pm\pi/2$, $\theta = \pi/2$) the heating potential is zero (see Fig. 6). This is due to the fact that only the lowest first-order mode is excited in the sphere.

In order to determine the effect of conductivity in the generation of hot spots, the case of a sphere of radius $A = 10$ cm but with a frequency-independent conductivity $\sigma = 1.0$ mho/m, increasingly lower than that of tissue as frequency increases, and relative di-

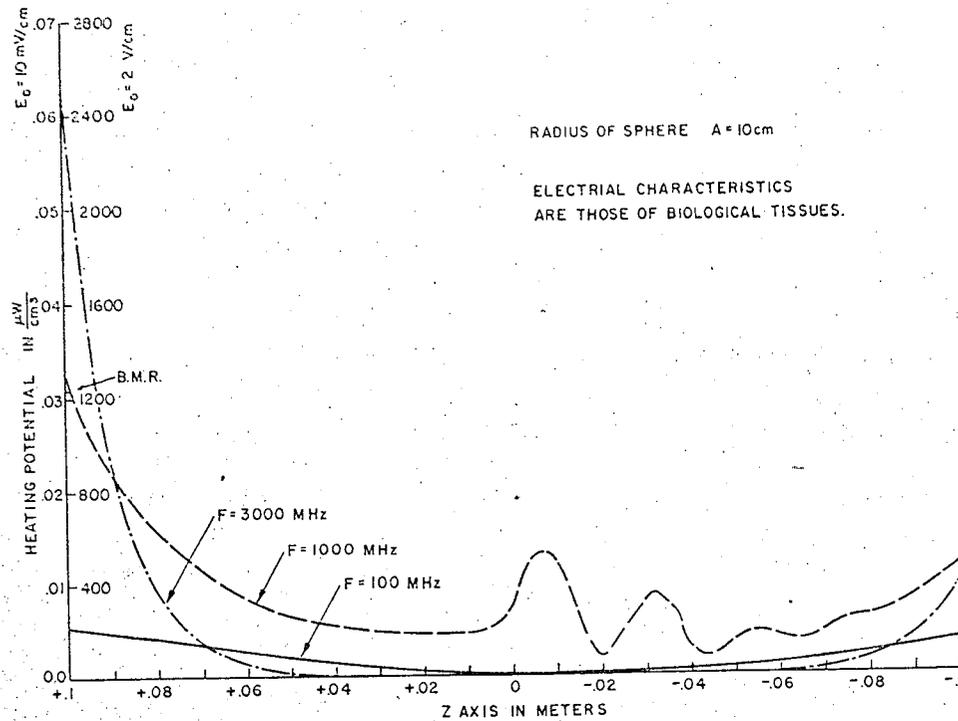


Fig. 4. Distribution of heating potential inside a sphere along the z axis.

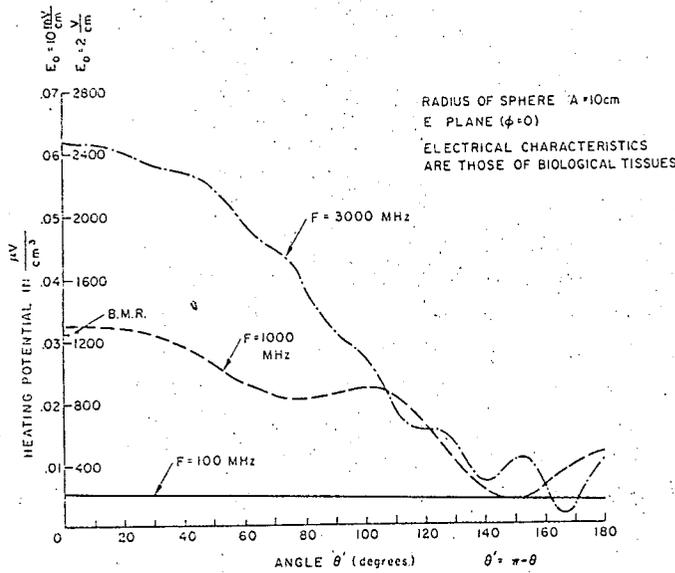


Fig. 5. Surface heating potential along the E plane.

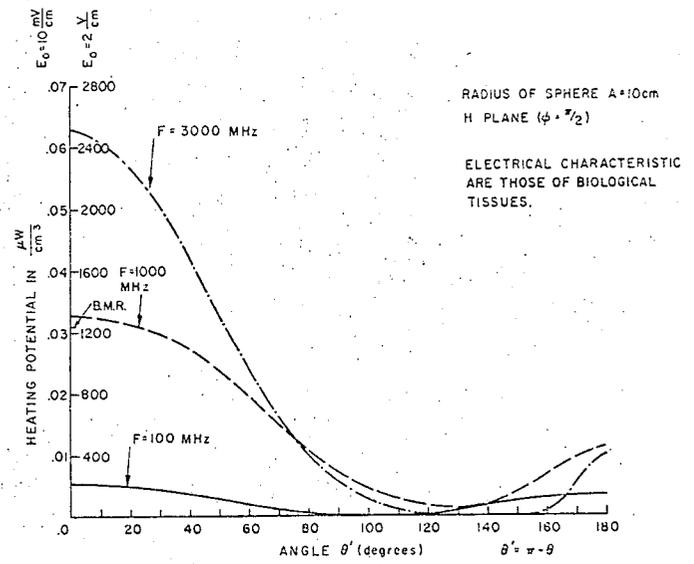


Fig. 6. Surface heating potential along the H plane.

electric constant $\epsilon_1 = 60$ was also examined. Unlike the previous case it was found that for $f > 1000$ MHz an intense hot spot is generated inside the sphere (see Figs. 7 and 8). This example serves to illustrate the fact that the generation of hot spots is strongly dependent upon conductivity.

The effect of the size is also an important parameter. A sphere of radius $A = 5$ cm was investigated (see Figs. 9 and 10). This would correspond to an idealized head of an infant. It was found that hot spots do occur inside in

the frequency region of $250 \text{ MHz} < f < 2800 \text{ MHz}$. The peak occurs at $f = 900$ MHz and is $\Psi = 0.143 \mu\text{W}/\text{cm}^3$, as compared to $\Psi = 0.026 \mu\text{W}/\text{cm}^3$ for the 10-cm sphere (Incident field is $10 \text{ mW}/\text{cm}^2$.)

The results of this investigation show that for a sphere of 10-cm radius having the electrical characteristics of common tissues, no hot spots occur inside the sphere. If, however, the ratio of the radius of the sphere to the penetration length is decreased by either decreasing the conductivity or decreasing the radius of

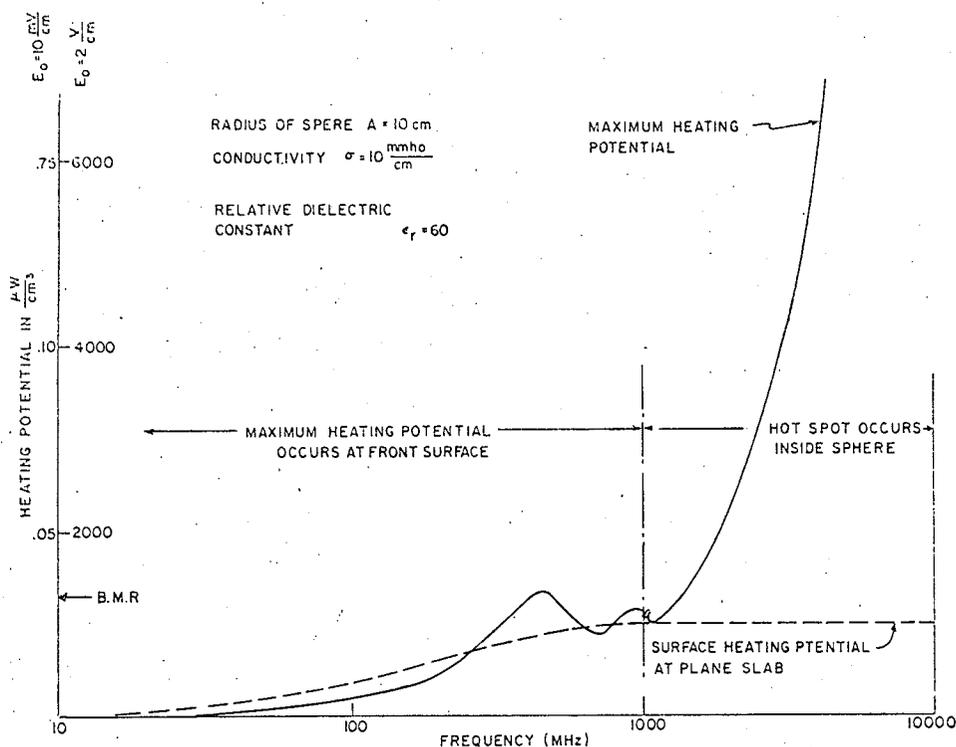


Fig. 7. Maximum heating potential of a sphere of 10-cm radius versus frequency.

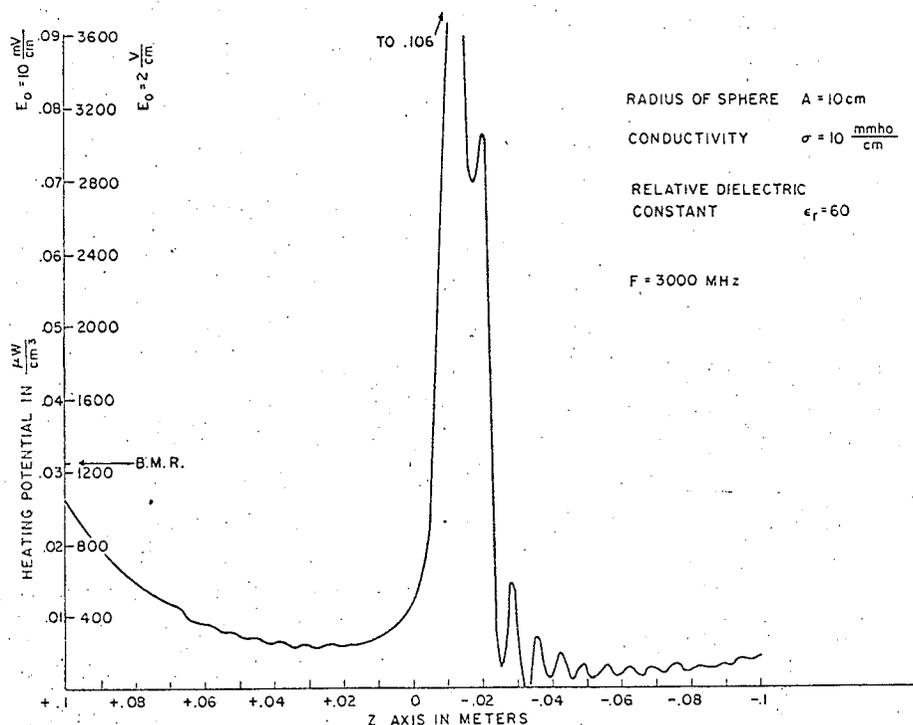


Fig. 8. Distribution of heating potential inside the sphere along the z axis.

the sphere, intense hot spots do occur inside the sphere. The idealized homogeneous model used in this paper does not account for the skin, fat, and bone layers; it also does not deal with the heat diffusion properties of the head such as conduction, respiration, and blood circulation. Its contribution lies in the fact that it distinctly demonstrates a new mechanism for possible

creation of hot spots in the head of an infant or small animal. The fact that, under the conditions stated above, the heating potential at the center can be sharply peaked at an order of magnitude higher than that of the plane slab strongly suggests that this is a potential problem area and that further work should be done to clarify this mechanism.

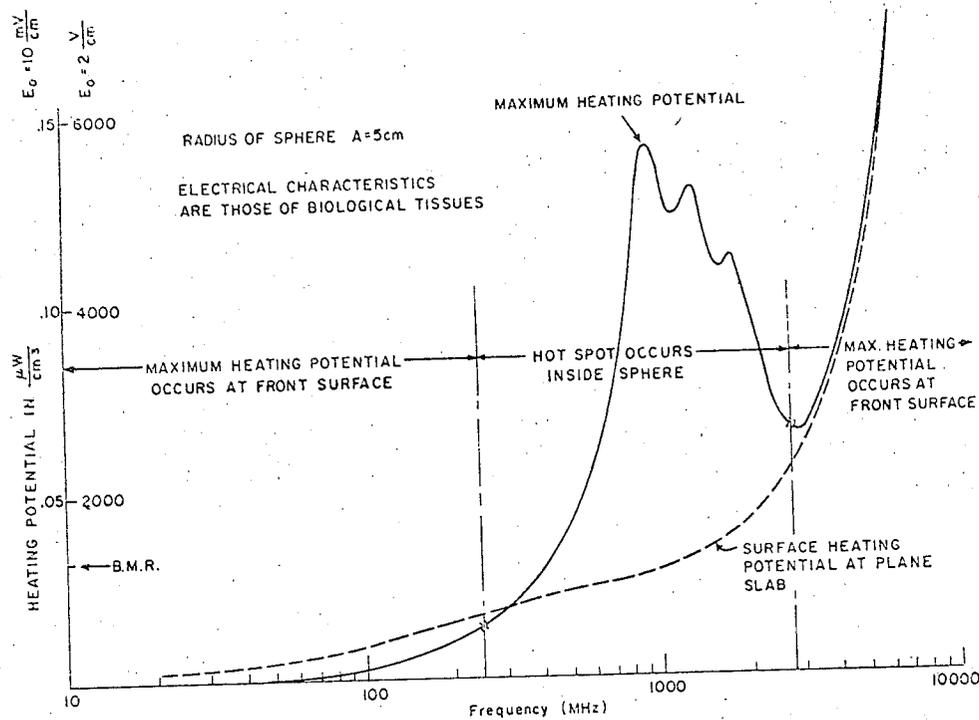


Fig. 9. Maximum heating potentials of a sphere of 5 cm having the same electrical characteristics as those of biological tissues versus frequency.

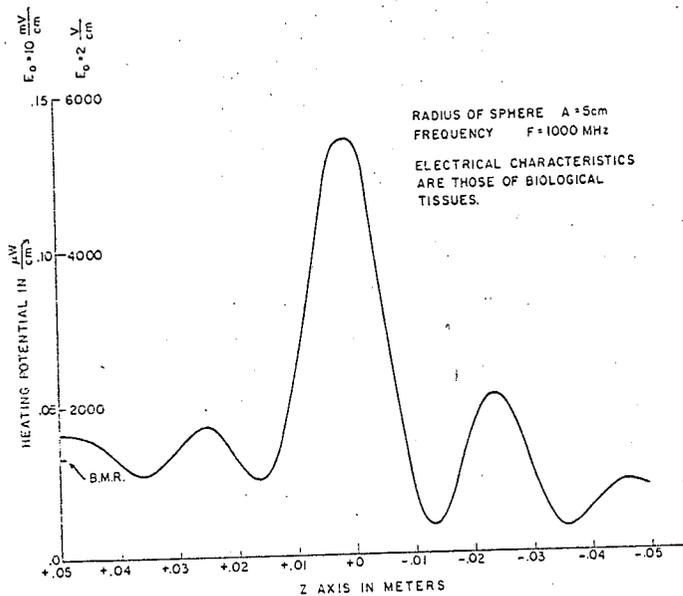


Fig. 10. Distribution of heating potential inside the sphere along the z axis.

It appears fortunate that the conductivity of tissues is high enough to prevent hot spots in large spheres and probably in the head of man. However, hot spots may occur in animals of smaller size. Thus experimental

results obtained with animals and conducted to learn about the biological hazards of microwaves do not necessarily pertain to man.

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