

The
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THE COMPUTATION OF RADIATION
AND SCATTERED ELECTROMAGNETIC FIELDS

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THE COMPUTATION OF RADIATION AND SCATTERED ELECTROMAGNETIC FIELDS

In this report we apply some of the results obtained earlier¹ to the practical numerical computation of radiation and scattered fields. The method we describe can be interpreted as a variational technique and its application is so far restricted to the problems we consider in this report. Nevertheless it has the definite advantage over other variational techniques that when applied repeatedly it yields a sequence of fields converging uniformly to the exact solution of the problem. This is a consequence of the facts established in reference 1. Also the method gives an estimate of the error at each step of the computation. As far as we know, convergence of other variational techniques to the solution of these problems has never been established rigorously. Nor is there available any estimate of the errors.

We shall consider the problem of solving Maxwell's equations in the infinite domain D exterior to a finite number of disjoint bounded regions V_i . We shall assume that the regions V_i are bounded by smooth surfaces Σ_i , as described in reference 1. Of the field we shall require that it satisfy the radiation condition and that the electric vector have a prescribed tangential component. As has been shown,¹ this problem has a unique solution. Given two source-free electromagnetic fields E_1, H_1 and E_2, H_2 in D , satisfying the radiation condition, we associate to this pair the number

$$[E_1, E_2] = \sum_i \int_{\Sigma_i} (E_1 \times n) \cdot (\bar{E}_2 \times n) d\sigma, \quad (1)$$

where \bar{E}_2 is the complex conjugate to E_2 , n is the unit vector normal to the surface Σ_i , and $d\sigma$ is the element of area of Σ_i . This quantity we shall call the mutual gauge of the fields E_1 and E_2 . When $E_1 = E_2 = E$ we shall call it simply the gauge of E . We note that the gauge of a field is a positive real number, and although as far as we know it does not have an immediate physical interpretation, it is a good measure of the "size" of the field. In fact, for any quantity x depending linearly and continuously on the values of the field and its derivatives at points away from the surfaces Σ_i we have that $|x| \leq A [E, E]^{\frac{1}{2}}$, where A is some constant depending on the nature of

the quantity x . If x depends quadratically on the values of E or its derivatives we have that $|x| \leq A [E, E]$. Such a quantity would be, for instance, the radiated energy. This follows from the fact that if E_n is a sequence of fields whose gauges tend to zero, then the fields themselves and all their derivatives tend to zero uniformly away from the surfaces Σ_i (see reference 1 paragraph 5). Now the practical significance of the gauge is that it can be computed from the tangential component of E along the Σ_i so that if E is a radiation or a scattered field to be determined, the gauge of E and the mutual gauge of E and other fields can be computed from the data of the problem.

Going back to our problem, suppose we are given the tangential component of a field E along the surfaces Σ_i and we want to approximate E by means of a linear combination of other known fields E_n . Then we compute the gauge of the difference between E and a linear combination of the E_n , that is,

$$\begin{aligned} \Delta &= [E - \sum \lambda_n E_n, E - \sum \lambda_n E_n] \\ &= [E, E] - \sum \{ \lambda_n [E_n, E] + \bar{\lambda}_n [E, E_n] \} \\ &\quad + \sum \lambda_n \bar{\lambda}_m [E_n, E_m] \end{aligned} \quad (2)$$

and minimize this expression. If λ_n^* are the minimizing values of the indeterminates, then Δ is stationary for these values of λ_n . If we set $E_a = \sum \lambda_n^* E_n$, then E_a is the linear combination of the E_n which approximates E best in the sense that the gauge of the difference $E - E_a$ is the smallest possible. Furthermore, it is readily seen that the mutual gauge of E_a and $E - E_a$ is zero, so that

$$[E, E_a] = [E_a, E_a] \quad ,$$

from which it follows that

$$\begin{aligned} [E - E_a, E - E_a] &= [E, E] - [E, E_a] - [E_a, E] + [E_a, E_a] \\ &= [E, E] - [E_a, E_a] \quad ; \end{aligned}$$

that is, the gauge of the error $E - E_a$ is equal to the gauge of E minus the gauge of E_a .

The choice of the fields E_n is arbitrary in theory but in practice it will depend on previous experience, or if the problem is that of computing a scattered field one may guess at possible electric current

distributions excited at the surface of the perfect conductors V_i by the incident field and take the corresponding electric fields as the E_n .

The practical problem of minimizing Δ in (2) reduces to solving a system of linear equations, but it can be approached directly by means of descent methods. For numerical computation the latter procedure is perhaps more adequate.

One can also proceed step by step, and this is a convenient way to proceed in case one has an infinite sequence of fields E_n available and one wishes to proceed with computing until a preassigned degree of accuracy is obtained.

Let E_n be the sequence of fields and E the given field. We define then the sequences \mathcal{E}_n , \mathcal{E}'_n as follows:

$$\begin{aligned}\mathcal{E}'_1 &= E_1 & \mathcal{E}_1 &= \mathcal{E}'_1 [\mathcal{E}'_1, \mathcal{E}'_1]^{-\frac{1}{2}} \\ \mathcal{E}_n &= E_n - \sum_{k=1}^{n-1} \mathcal{E}_k [E_n, \mathcal{E}_k] & \mathcal{E}'_n &= \mathcal{E}'_n [\mathcal{E}'_n, \mathcal{E}'_n]^{-\frac{1}{2}}.\end{aligned}$$

Then the approximating fields are

$$E_{a_n} = \sum_{k=1}^n \mathcal{E}_k [E, \mathcal{E}_k] \quad (3)$$

It is readily seen that what we have done here is orthogonalize the fields E_n with respect to the gauge. Since

$$[\mathcal{E}_n, \mathcal{E}_m] = \delta_{nm}, \quad \delta_{nm} = \begin{cases} 1 & n=m \\ 0 & n \neq m \end{cases}$$

the gauge of E_{a_n} is

$$[E_{a_n}, E_{a_n}] = \sum_{k=1}^n | [E, \mathcal{E}_k] |^2.$$

There arises the question of whether the E_{a_n} in (3) will approximate E to any arbitrarily close degree by taking n sufficiently large. The answer to this question is affirmative if one takes the E_n to be the fields of magnetic dipoles, three of them, mutually perpendicular, in each V_i , and all their partial derivatives. Since these fields satisfy Maxwell's equation, their derivatives will not be linearly independent

and it will be sufficient, for example, to take all their derivatives with respect to x and y and all such derivatives of their first derivatives with respect to z .

BIBLIOGRAPHY

1. The Multipole Expansion of Radiation Fields, technical report 478-16, 15 August 1953, Antenna Laboratory, The Ohio State University Research Foundation; prepared under Contract AF18(600)-88, Air Research and Development Command, Wright Air Development Center, Wright-Patterson Air Force Base, Ohio.

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